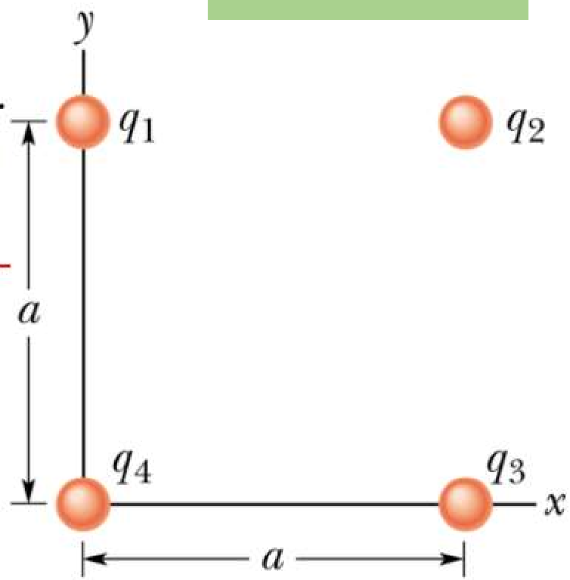
GPN2-L02

Problem 1

In the figure, the four particles form a square of edge length $a = 5cm$ and have charges $q_1 = +10$ nC, $q_2 = -20$ nC , $q_3 = +20$ nC, and $q_4 = -10$ nC. In unit-vector notation, what net electric field do the particles produce at the square's center? (02小題)



E_x=______ N/C

01: **ANS:=0**

E_y=______ N/C

02: **ANS:=1.02E5**

The x component of the electric field at the center of the square is given by

$$E_x = \frac{1}{4\pi\varepsilon_0}\left[\frac{|q_1|}{(a/\sqrt{2})^2}+\frac{|q_2|}{(a/\sqrt{2})^2}-\frac{|q_3|}{(a/\sqrt{2})^2}-\frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45°$$
$$=\frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}(|q_1|+|q_2|-|q_3|-|q_4|)\frac{1}{\sqrt{2}}$$
$$=0.$$

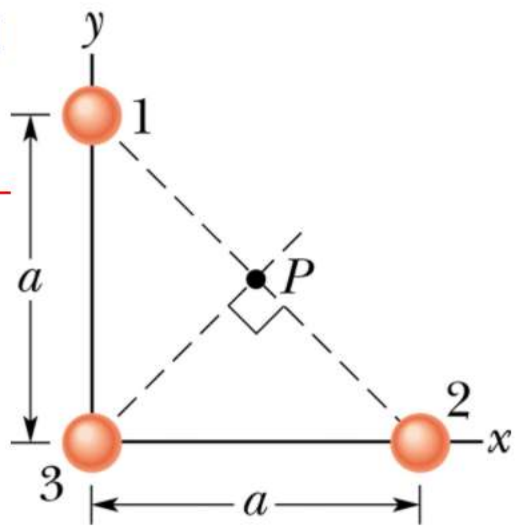
Similarly, the y component of the electric field is

$$E_y = \frac{1}{4\pi\varepsilon_0}\left[-\frac{|q_1|}{(a/\sqrt{2})^2}+\frac{|q_2|}{(a/\sqrt{2})^2}+\frac{|q_3|}{(a/\sqrt{2})^2}-\frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45°$$
$$=\frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}(-|q_1|+|q_2|+|q_3|-|q_4|)\frac{1}{\sqrt{2}}$$
$$=\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2.0\times10^{-8}\ \mathrm{C})}{(0.050\ \mathrm{m})^2/2}\frac{1}{\sqrt{2}}=1.02\times10^5\ \mathrm{N/C}.$$

Thus, the electric field at the center of the square is $\vec{E}=E_y\hat{j}=(1.02\times10^5\ \mathrm{N/C})\hat{j}$.

Problem 2

In the figure, the three particles are fixed in place and have charges $q_1 = q_2 = +e$ and $q_3 = +2e$. Distance $a = 6\ \mu$m. What are the magnitude of the net electric field at point P due to the particles? (01小題)



the net electric field $E =$______ N/C

03: **ANS:=<u>160</u>**

Solution:

13. By symmetry we see the contributions from the two charges $q_1 = q_2 = +e$ cancel each other, and we simply use Eq. 22-3 to compute magnitude of the field due to $q_3 = +2e$.

(a) The magnitude of the net electric field is

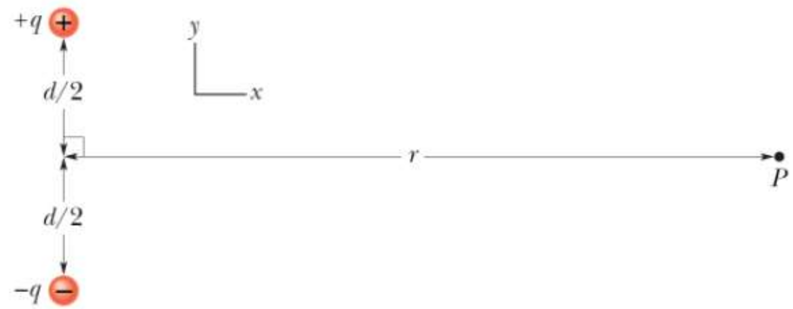
$$|\vec{E}_{\text{net}}| = \frac{1}{4\pi\varepsilon_0}\frac{2e}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{2e}{(a/\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{4e}{a^2}$$

$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4(1.60\times10^{-19}\ \text{C})}{(6.00\times10^{-6}\ \text{m})^2} = 160\ \text{N/C}.$$

(b) This field points at 45.0°, counterclockwise from the x axis.

Problem 3

The figure shows an electric dipole. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the dipole's electric field at point P, located at distance $r << d$? (02小題)



(a)magnitude of electric field, |E|=_____ [epsilon_0,q,d,r]

04: **ANS:=(1/(4*pi*epsilon_0))*(q*d/r**3)**

(b)$1 = \hat{i}; 2 = -\hat{i}; 3 = \hat{j}; 4 = -\hat{j};$ the direction=____

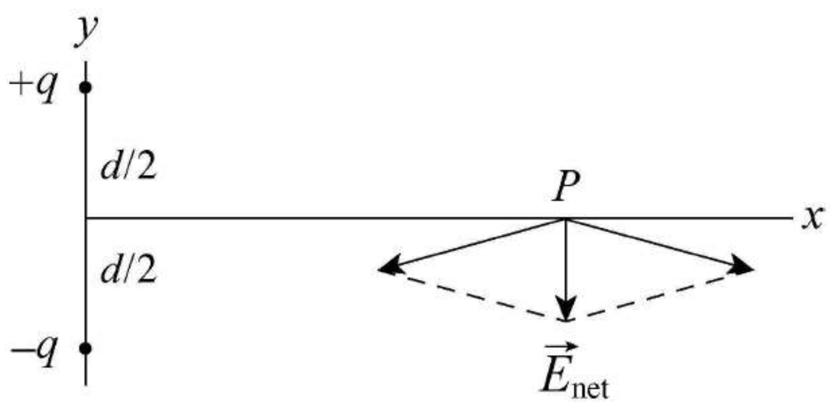
05: **ANS:=4**

$$\left|\vec{E}_{\text{net}}\right| = 2E_1 \sin\theta = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{(d/2)^2 + r^2}\right]\frac{d/2}{\sqrt{(d/2)^2 + r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left[(d/2)^2 + r^2\right]^{3/2}}$$

For $r \gg d$, we write $[(d/2)^2 + r^2]^{3/2} \approx r^3$ so the expression above reduces to

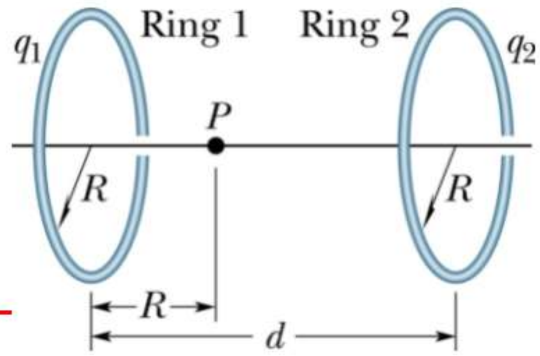
$$|\vec{E}_{\text{net}}| \approx \frac{1}{4\pi\varepsilon_0}\frac{qd}{r^3}.$$

(b) From the figure, it is clear that the net electric field at point P points in the $-\hat{\text{j}}$ direction, or $-90°$ from the $+x$ axis.



Problem 4

The figure shows two parallel nonconducting rings with their central axes along a common line. Ring 1 has uniform charge $q1$ and radius R; ring 2 has uniform charge $q2$ and the same radius R. The rings are separated by distance $d = 3R$. The net electric field at point P on the common line, at distance R from ring 1, is zero. What is the ratio $\frac{q_1}{q_2}$? (01小題)



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the ratio $\frac{q_1}{q_2}$ =____

06: **ANS:=0.506**

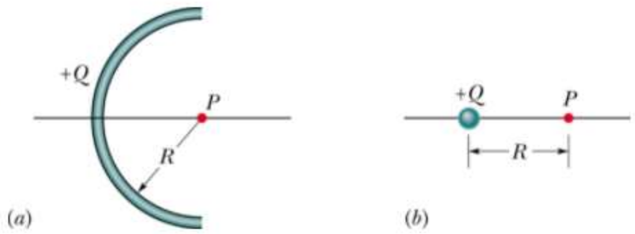
assuming both charges are positive. At P, we have

$$E_{\text{left ring}} = E_{\text{right ring}} \Rightarrow \frac{q_1 R}{4\pi\varepsilon_0 \left(R^2 + R^2\right)^{3/2}} = \frac{q_2 (2R)}{4\pi\varepsilon_0 [(2R)^2 + R^2]^{3/2}}$$

$$\frac{q_1}{q_2} = 2\left(\frac{2}{5}\right)^{3/2} \approx 0.506.$$

Problem 5

The figure(a) shows a nonconducting rod with a uniformly distributed charge $+Q$. The rod forms a half-circle with radius R and produces an electric field of magnitude E_{arc} at its center of curvature P. If the arc is collapsed to a point at distance R from P (the figure(b)), by what factor is the magnitude of the electric field at P multiplied? (01小題)



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the factor= $\frac{E_{particle}}{E_{arc}}$ =____

07: **ANS:=1.57**

$$\lambda = Q/L. \qquad E_{arc} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}.$$

$$L = r\theta \qquad E_{arc} = \frac{2(Q/L)\sin(\theta/2)}{4\pi\varepsilon_0 r} = \frac{2(Q/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 r} = \frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta}.$$

$$\frac{E_{particle}}{E_{arc}} = \frac{Q/4\pi\varepsilon_0 R^2}{2Q\sin(\theta/2)/4\pi\varepsilon_0 R^2\theta} = \frac{\theta}{2\sin(\theta/2)}.$$

With $\theta = \pi$, we have $$\frac{E_{particle}}{E_{arc}} = \frac{\pi}{2} \approx 1.57.$$

Problem 6

An electron is released from rest in a uniform electric field of magnitude 2×10^4 N/C. Calculate the acceleration of the electron. (Ignore gravitation.) (01小題)

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the acceleration=___m/s^2

08: **ANS:=<u>3.51E15</u>**

The magnitude of the force acting on the electron is $F = eE$, where E is the magnitude of the electric field at its location. The acceleration of the electron is given by Newton's second law:

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{(1.60 \times 10^{-19}\ \text{C})(2.00 \times 10^4\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}} = 3.51 \times 10^{15}\ \text{m/s}^2 .$$

Problem 7

A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0 cm away, in a time 1.5×10^{-8} s. (a) What is the speed of the electron as it strikes the second plate? (b) What is the magnitude of the electric field $|\vec{E}|$? (02小題)

(a)v=____ m/s

09: **ANS:=2.7E6**

(b)$|\vec{E}|$=_____ N/C

10: **ANS:=1000**

(a) We use $\Delta x = v_{avg}t = vt/2$:

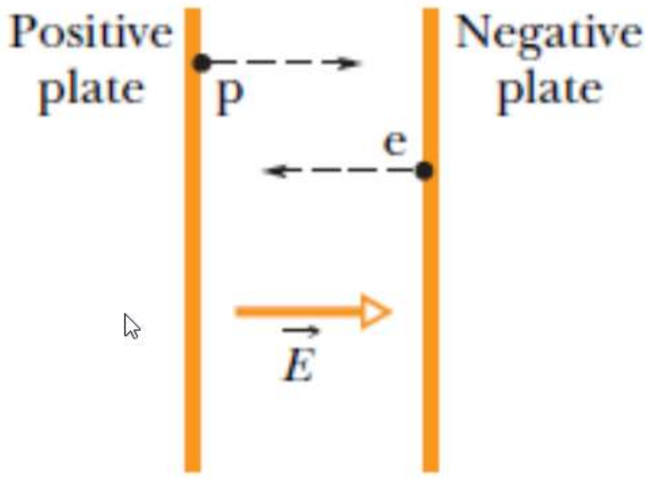
$$v = \frac{2\Delta x}{t} = \frac{2(2.0\times10^{-2}\,\text{m})}{1.5\times10^{-8}\,\text{s}} = 2.7\times10^{6}\ \text{m/s}.$$

(b) We use $\Delta x = \frac{1}{2}at^2$ and $E = F/e = ma/e$:

$$E = \frac{ma}{e} = \frac{2\Delta x m}{et^2} = \frac{2(2.0\times10^{-2}\,\text{m})(9.11\times10^{-31}\,\text{kg})}{(1.60\times10^{-19}\,\text{C})(1.5\times10^{-8}\,\text{s})^2} = 1.0\times10^{3}\ \text{N/C}.$$

Problem 8

Two large parallel copper plates are 5.0 cm apart and have a uniform electric field between them as depicted in the figure. An electron is released from the negative plate at the same time that a proton is released from the positive plate. Neglect the force of the particles on each other and find their distance from the positive plate when they pass each other. (Does it surprise you that you need not know the electric field to solve this problem?) The mass of electron and proton are 9.1×10^{-31} kg and 1.67×10^{-27} kg, respectively. (01小題)



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the distance from the positive plate=________ m

11: **ANS:=2.7E-5**

$$a_e = -eE/m_e, \qquad a_p = eE/m_p$$

$$\tfrac{1}{2}a_p t^2 = L + \tfrac{1}{2}a_e t^2.$$

$$x = \frac{a_p}{a_p - a_e}L = \frac{eE/m_p}{(eE/m_p)+(eE/m_e)}L = \left(\frac{m_e}{m_e + m_p}\right)L$$

$$= \left(\frac{9.11\times10^{-31}\text{kg}}{9.11\times10^{-31}\text{kg}+1.67\times10^{-27}\text{kg}}\right)(0.050\text{m})$$

$$= 2.7\times10^{-5}\text{ m.}$$

Problem 9

An electric dipole consisting of charges of magnitude 1.50 nC separated by $6.2\mu m$ is in an electric field of strength $1100N/C$. What is the magnitude of the electric dipole moment? (01小題)

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the magnitude of the electric dipole moment, $p =$______$C \cdot m$

12: **ANS:=<u>9.3E-15</u>**

(a) The magnitude of the dipole moment is

$$p = qd = (1.50\times10^{-9}\ \text{C})(6.20\times10^{-6}\ \text{m}) = 9.30\times10^{-15}\ \text{C}\cdot\text{m}.$$

(b) Following the solution to part (c) of Sample Problem 22-5, we find

$$U(180°) - U(0) = 2pE = 2(9.30\times10^{-15}\text{C}\cdot\text{m})(1100\ \text{N/C}) = 2.05\times10^{-11}\ \text{J}.$$

Problem 10

Find an expression for the oscillation frequency f of an electric dipole of dipole moment p and rotational inertia I for small amplitudes of oscillation about its equilibrium position in a uniform electric field of magnitude E. (01小題)

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frequency f=____

13: **ANS:=(1/(2*pi))*sqrt((p*E)/I)**

$$\tau = -pE\sin\theta$$

$$\tau \approx -pE\theta.$$

$$\omega^2 = \frac{\kappa}{I} = \frac{pE}{I}$$

torsion constant $\kappa = pE$.

where I is the rotational inertia of the dipole.

The frequency is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}$.