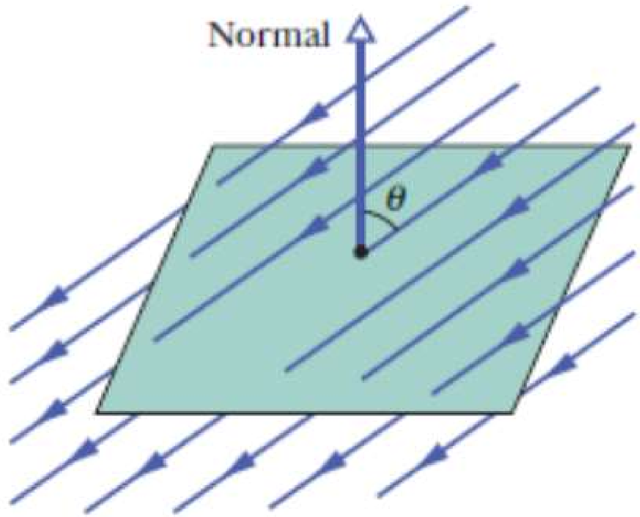
**GPN2-L03**

## Problem 1

The square surface shown in the figure measures 3.2 mm on each side. It is immersed in a uniform electric field with magnitude E=1800N/C and with field lines at an angle of $\theta = 35^\circ$ with a normal to the surface, as shown. Take that normal to be directed "outward," as though the surface were one face of a box. Calculate the electric flux through the surface. (01小題)

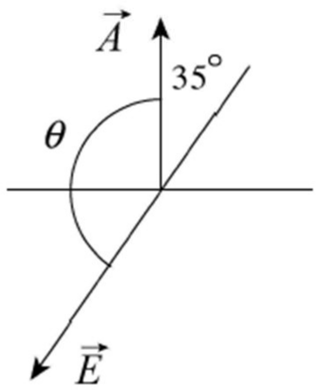



the electric flux=________N/C·m$^2$

01: **ANS:=-1.5E-2**

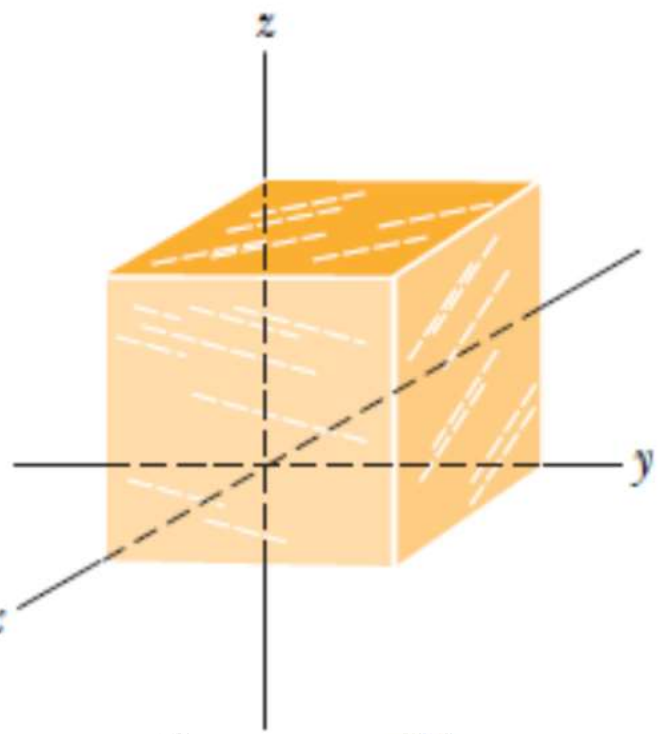
1. The vector area $\vec{A}$ and the electric field $\vec{E}$ are shown on the diagram below. The angle $\theta$ between them is $180° - 35° = 145°$, so the electric flux through the area is

$$\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta = (1800\ \text{N/C})(3.2\times10^{-3}\ \text{m})^2\cos145° = -1.5\times10^{-2}\ \text{N}\cdot\text{m}^2/\text{C}.$$



**Problem 1**

The cube in the figure has edge length 1.40 m and is oriented as shown in a region of uniform electric field. Find the electric flux through the right face if the electric field, in newtons per coulomb, is given by (a) $6\mathbf{i}$ , (b) $-2\mathbf{j}$ , and (c) $-3\mathbf{i}+4\mathbf{k}$ . (d) What is the total flux through the cube for each field? (04小題)




(a) the electric flux=________N/C·$m^2$

02: **ANS:=0**

(b) the electric flux=________N/C·$m^2$

03: **ANS:=-3.92**

(c) the electric flux=________N/C·$m^2$

04: **ANS:=0**

(d) the total flux=________N/C·$m^2$

05: **ANS:=0**

$$\Phi = \vec{E}\cdot\vec{A}\text{, where } \vec{A} = A\hat{j} = (1.40\text{m})^2\,\hat{j}.$$

(a) $\Phi = (6.00\ \text{N/C})\hat{i}\cdot(1.40\ \text{m})^2\ \text{j} = 0.$

(b) $\Phi = (-2.00\ \text{N/C})\hat{j}\cdot(1.40\ \text{m})^2\ \text{j} = -3.92\ \text{N}\cdot\text{m}^2/\text{C}.$

(c) $\Phi = \left[(-3.00\ \text{N/C})\hat{i} + (400\,\text{N/C})\text{k}\right]\cdot(1.40\ \text{m})^2\,\hat{j} = 0.$

(d) The total flux of a uniform field through a closed surface is always zero.

**Problem 2**

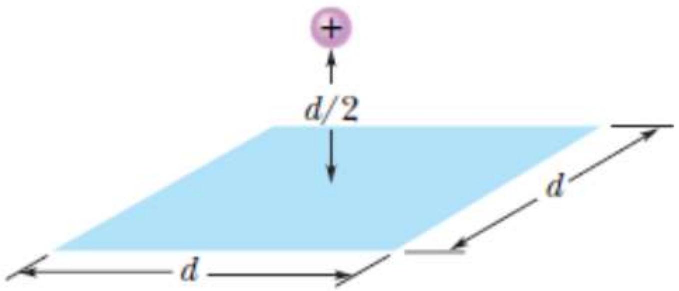
A point charge of 1.8 $\mu$C is at the center of a cubical Gaussian surface 55 cm on edge. What is the net electric flux through the surface? (01小題)

the net electric flux=________N/C$\cdot m^2$

06: **ANS:=2E5**

$$\text{Gauss' law: } \varepsilon_0\Phi = q, \quad \Phi = \frac{q}{\varepsilon_0} = \frac{1.8\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 2.0\times10^5\ \text{N}\cdot\text{m}^2/\text{C}.$$

In the figure, a proton is a distance $\frac{d}{2}$ directly above the center of a square of side $d$. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge $d$.) (01小題)



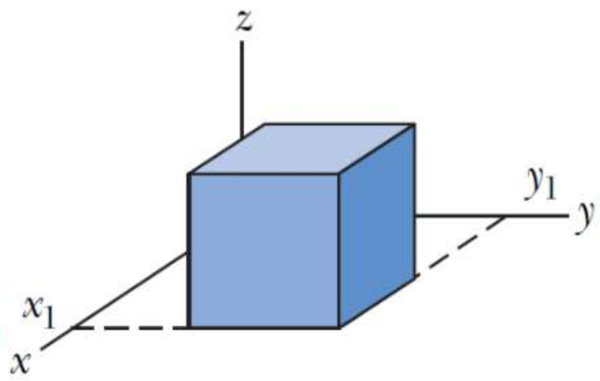

the magnitude of the electric flux=________N/C$\cdot m^2$

07: **ANS:=3.01E-9**

$$\Phi = \frac{q}{6\varepsilon_0} = \frac{1.6\times10^{-19}\ \text{C}}{6(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.01\times10^{-9}\ \text{N}\cdot\text{m}^2/\text{C}.$$

**Problem 3**

The figure shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m, with one corner at $x_1 = 5.00$ m, $y_1 = 4.00$ m. The cube lies in a region where the electric field vector is given by $\vec{E} = -3\hat{i} - 4y^2\hat{j} + 3\hat{k}$ N/C, with y in meters. What is the net charge contained by the cube? (01小題)

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the net charge=_____ C

08: **ANS:=-1.7E-9**

The face of the cube located at $y = 4.00$ has area $A = 4.00\ \text{m}^2$ $+\hat{j}$

has a "contribution" to the flux equal to

$$E_{\text{non-constant}} A = (-4)(4^2)(4) = -256\ \text{N}\cdot\text{m/C}^2.$$

The face of the cube located at $y = 2.00$ m $-\hat{j}$ direction

$$-E_{\text{non-constant}} A = -(-4)(2^2)(4) = 64\ \text{N}\cdot\text{m/C}^2.$$

$$\Phi = (-256 + 64)\ \text{N}\cdot\text{m/C}^2 = -192\ \text{N}\cdot\text{m/C}^2.$$

$$q_{\text{enc}} = \varepsilon_0 \Phi = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(-192\ \text{N}\cdot\text{m}^2/\text{C}) = -1.70\times10^{-9}\,\text{C}.$$

**Problem 3**

A uniformly charged conducting sphere of 1.2 m diameter has a surface charge density of 8.1 $\mu C/m^2$ . (a) Find the net charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere? (02小題)

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(a) the net charge=________C

09: **ANS:=3.66E-5**

(b) the electric flux=________N/C·$m^2$

10: **ANS:=4.1E6**

(a) The charge on the surface of the sphere is the product of the surface charge density $\sigma$ and the surface area of the sphere (which is $4\pi r^2$, where $r$ is the radius). Thus,

$$q = 4\pi r^2 \sigma = 4\pi \left(\frac{1.2 \text{ m}}{2}\right)^2 \left(8.1\times10^{-6} \text{ C/m}^2\right) = 3.7\times10^{-5} \text{ C}.$$

(b) We choose a Gaussian surface in the form of a sphere, concentric with the conducting sphere and with a slightly larger radius. The flux is given by Gauss's law:

$$\Phi = \frac{q}{\varepsilon_0} = \frac{3.66\times10^{-5} \text{ C}}{8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} = 4.1\times10^6 \text{ N}\cdot\text{m}^2/\text{C}.$$

## Problem 4

An isolated conductor of arbitrary shape has a net charge of $+10 \times 10^{-6}$ C. Inside the conductor is a cavity within which is a point charge $q = 3 \times 10^{-6}$ C.What is the charge (a) on the cavity wall and (b) on the outer surface of the conductor? (02小題)

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(a) the charge on the cavity wall =________C

11: **ANS:=-3E-6**

(b) the charge on the outer surface of the conductor=________C

12: **ANS:=1.3E-5**

(a) Consider a Gaussian surface that is completely within the conductor and surrounds the cavity. Since the electric field is zero everywhere on the surface, the net charge it encloses is zero. The net charge is the sum of the charge $q$ in the cavity and the charge $q_w$ on the cavity wall, so $q + q_w = 0$ and $q_w = -q = -3.0 \times 10^{-6}$C.

(b) The net charge $Q$ of the conductor is the sum of the charge on the cavity wall and the charge $q_s$ on the outer surface of the conductor, so $Q = q_w + q_s$ and

$$q_s = Q - q_\omega = \left(10\times10^{-6}\ \text{C}\right) - \left(-3.0\times10^{-6}\ \text{C}\right) = +1.3\times10^{-5}\ \text{C}.$$

## Problem 4

An infinite line of charge produces a field of magnitude $4.5 \times 10^4$ N/C at a distance of 2.0 m. Calculate the linear charge density. (01小題)

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the linear charge density=________C/m

13: **ANS:=<u>5E1-6</u>**

The magnitude of the electric field produced by a uniformly charged infinite line is $E = \lambda/2\pi\varepsilon_0 r$, where $\lambda$ is the linear charge density and $r$ is the distance from the line to the point where the field is measured. See Eq. 23-12. Thus,

$$\lambda = 2\pi\varepsilon_0 Er = 2\pi\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(4.5\times10^{4}\ \mathrm{N/C}\right)\left(2.0\ \mathrm{m}\right) = 5.0\times10^{-6}\ \mathrm{C/m}.$$

## Problem 5

帶電同軸電纜線的電場問題：Two long, charged, thin-walled, concentric cylindrical shells have radii of 3.0 and 6.0 cm. The charge per unit length is $5.0 \times 10^{-6}$ C/m on the inner shell and $-7.0 \times 10^{-6}$ C/m on the outer shell. What are the (a) magnitude $E$ and (b) direction (radially inward or outward) of the electric field at radial distance $r = 4.0$ cm? What are (c) $E$ and (d) the direction at $r = 8.0$ cm? (04小題)

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(a) $|E|$ =______N/C

14: **ANS:=2.25E6**

(b) 1=outward, 2=inward

15: **ANS:=1**

(c) $|E|$ =______N/C

16: **ANS:=4.45E5**

(d) 1=outward, 2=inward

17: **ANS:=2**

We denote the inner and outer cylinders with subscripts $i$ and $o$, respectively.

(a) Since $r_i < r = 4.0 \text{ cm} < r_o$,

$$E(r) = \frac{\lambda_i}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\ \text{C/m}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.0\times10^{-2}\ \text{m})} = 2.3\times10^{6}\ \text{N/C}.$$

(b) The electric field $\vec{E}(r)$ points radially outward.

(c) Since $r > r_o$,

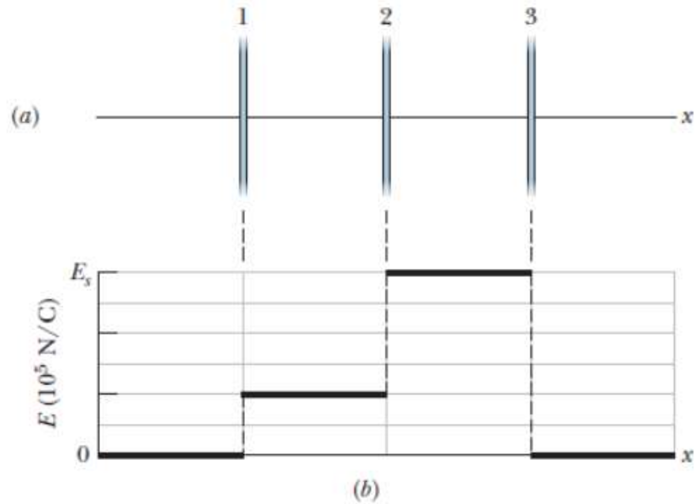
$$E(r = 8.0\ \text{cm}) = \frac{\lambda_i + \lambda_o}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\ \text{C/m} - 7.0\times10^{-6}\ \text{C/m}}{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8.0\times10^{-2}\ \text{m})} = -4.5\times10^{5}\ \text{N/C},$$

or $|E(r = 8.0\ \text{cm})| = 4.5\times10^{5}$ N/C.

(d) The minus sign indicates that $\vec{E}(r)$ points radially inward.

## Problem 6

The figure(a) shows three plastic sheets that are large, parallel, and uniformly charged. The figure(b) gives the component of the net electric field along an x axis through the sheets. The scale of the vertical axis is set by $E_s = 6 \times 10^5$ N/C. What is the ratio of the charge density on sheet 3 to that on sheet 2? (01小題)




the ratio $\frac{\sigma_3}{\sigma_2}$ =_______

18: **ANS:=-1.5**

In the region between sheets 1 and 2, the net field is $E_1 - E_2 + E_3 = 2.0 \times 10^5$ N/C .

In the region between sheets 2 and 3, the net field is at its greatest value:

$$E_1 + E_2 + E_3 = 6.0 \times 10^5 \text{ N/C} .$$

The net field vanishes in the region to the right of sheet 3, where $E_1 + E_2 = E_3$ . We note the implication that $\sigma_3$ is negative (and is the largest surface-density, in magnitude). These three conditions are sufficient for finding the fields:

$$E_1 = 1.0 \times 10^5 \text{ N/C},\ E_2 = 2.0 \times 10^5 \text{ N/C},\ E_3 = 3.0 \times 10^5 \text{ N/C} .$$

From Eq. 23-13, we infer (from these values of $E$)

$$\frac{|\sigma_3|}{|\sigma_2|} = \frac{3.0 \times 10^5 \text{ N/C}}{2.0 \times 10^5 \text{ N/C}} = 1.5 .$$

Recalling our observation, above, about $\sigma_3$, we conclude $\frac{\sigma_3}{\sigma_2} = -1.5$ .

## Problem 6

A square metal plate of edge length 8.0 cm and negligible thickness has a total charge of $6 \times 10^{-6}$ C.(a) Estimate the magnitude $E$ of the electric field just off the center of the plate (at, say, a distance of 0.50 mm from the center) by assuming that the charge is spread uniformly over the two faces of the plate. (b) Estimate E at a distance of 30 m (large relative to the plate size) by assuming that the plate is a point charge. In SI unit. (02小題)

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(a) $E$=________N/C

19: **ANS:=5.3E7**

(b) $E$=________N/C

20: **ANS:=60**

The charge is distributed uniformly over both sides of the original plate,

$$\sigma = \frac{q}{2A} = \frac{6.0\times10^{-6}\ \text{C}}{2(0.080\ \text{m})^2} = 4.69\times10^{-4}\ \text{C/m}^2.$$

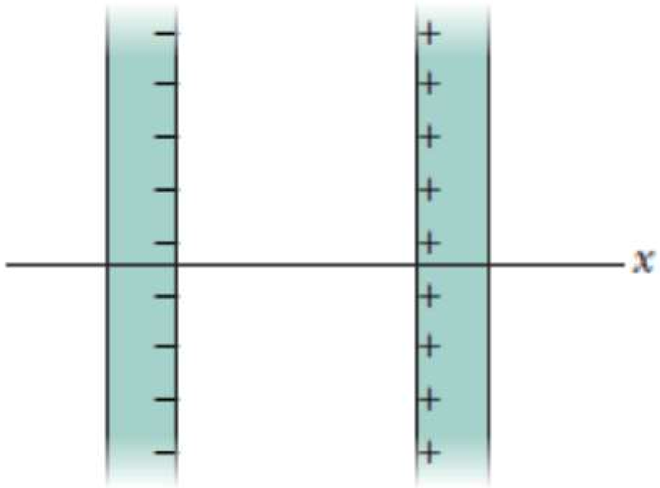
$$E = \frac{\sigma}{\varepsilon_0} = \frac{4.69\times10^{-4}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 5.3\times10^7\ \text{N/C}.$$

(b) At a point far away from the plate, the electric field is nearly that of a point particle with charge equal to the total charge on the plate. The magnitude of the field is $E = q/4\pi\varepsilon_0 r^2 = kq/r^2$, where $r$ is the distance from the plate. Thus,

$$E = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(6.0\times10^{-6}\ \text{C}\right)}{(30\ \text{m})^2} = 60\ \text{N/C}.$$

**Problem 7**

In the figure, two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have excess surface charge densities of opposite signs and magnitude $7 \times 10^{-22}$ C/$m^2$ . In unit-vector notation, what is the electric field at points (a) to the left of the plates, (b) to the right of them, and (c) between them? (03小題)




(a) the electric field=________N/C

21: **ANS:=0**

(b) the electric field=________N/C

22: **ANS:=0**

(c) the electric field=________N/C

23: **ANS:=-7.91E-11**

(a) To the left of the plates:

$$\vec{E} = (\sigma / 2\varepsilon_0)(-\hat{i}) \text{ (from the right plate)} + (\sigma / 2\varepsilon_0)\hat{i} \text{ (from the left one)} = 0.$$

(b) To the right of the plates:

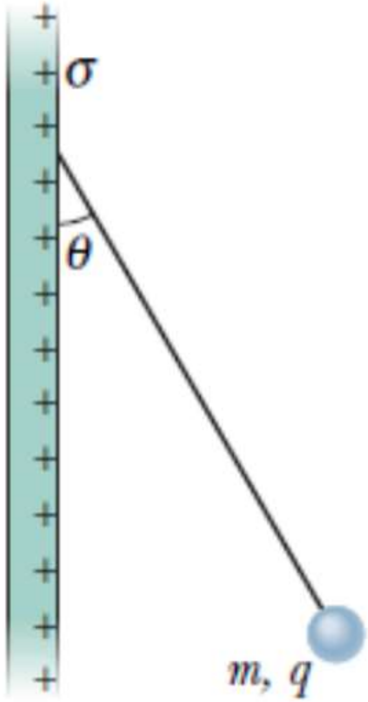
$$\vec{E} = (\sigma / 2\varepsilon_0)\hat{i} \text{ (from the right plate)} + (\sigma / 2\varepsilon_0)(-\hat{i}) \text{ (from the left one)} = 0.$$

(c) Between the plates:

$$\vec{E} = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{i}) + \left(\frac{\sigma}{2\varepsilon_0}\right)(-i) = \left(\frac{\sigma}{\varepsilon_0}\right)(-\hat{i}) = -\left(\frac{7.00\times10^{-22}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\right)i = (-7.91\times10^{-11}\ \text{N/C})\hat{i}.$$

## Problem 8

In the figure, a small, nonconducting ball of mass $m = 1$ mg and charge $q = 2 \times 10^{-8}$ C (distributed uniformly through its volume) hangs from an insulating thread that makes an angle $\theta = 30^\circ$ with a vertical, uniformly charged nonconducting sheet (shown in cross section). Considering the gravitational force on the ball and assuming the sheet extends far vertically and into and out of the page, calculate the surface charge density $\sigma$ of the sheet. (01小題)




the surface charge density $\sigma$=__________C/$m^2$

24: **ANS:=5.007e-9**

$$qE - T\sin\theta = 0$$

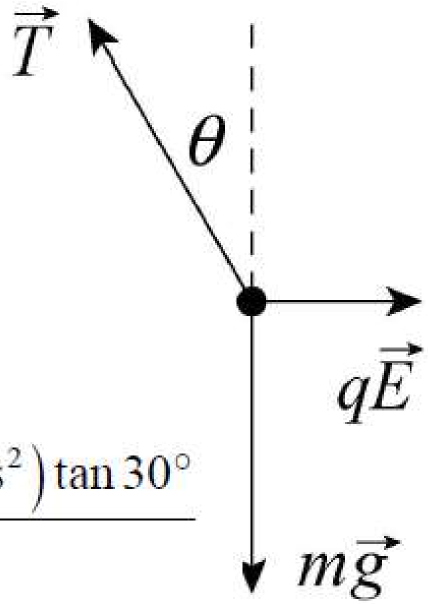
$$T\cos\theta - mg = 0\,.$$

$$qE = mg\tan\theta.$$

$$E = \sigma/2\varepsilon_0,$$

$$\frac{q\sigma}{2\varepsilon_0} = mg\tan\theta$$

$$\sigma = \frac{2\varepsilon_0 mg\tan\theta}{q}$$




$$= \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N.m}^2\right)\left(1.0\times10^{-6}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)\tan 30^\circ}{2.0\times10^{-8}\ \text{C}}$$

$$= 5.0\times10^{-9}\ \text{C/m}^2.$$

## Problem 9

An unknown charge sits on a conducting solid sphere of radius 10 cm. If the electric field 15 cm from the center of the sphere has the magnitude $3.0 \times 10^3$ N/C and is directed radially inward, what is the net charge on the sphere? (01小題)

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$q =$__________C

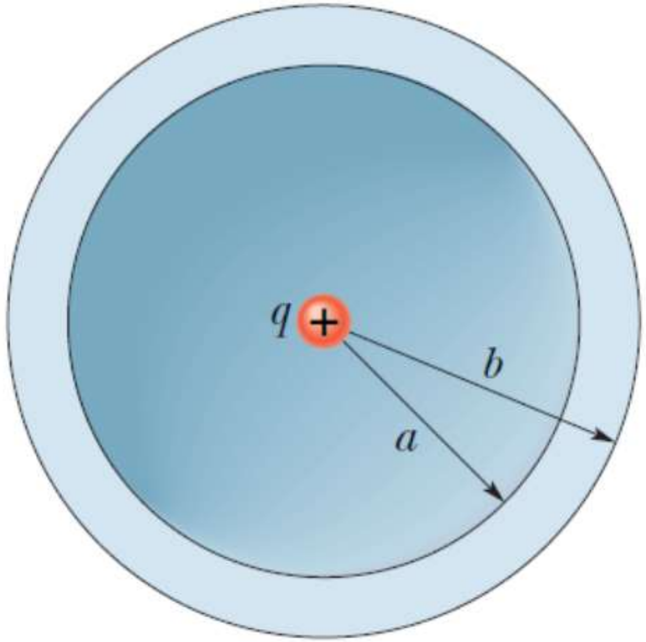
25: **ANS:=-7.508e-9**

$$|q| = 4\pi\varepsilon_0 r^2 E = \frac{(0.15\text{ m})^2 (3.0\times10^3\text{ N/C})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 7.5\times10^{-9}\text{ C}.$$

The field points inward, toward the sphere center, so the charge is negative, i.e., $q = -7.5\times10^{-9}$C.

## Problem 10

In the figure, a nonconducting spherical shell of inner radius $a = 2.00$ cm and outer radius $b = 2.40$ cm has (within its thickness) a positive volume charge density $\rho = A/r$, where $A$ is a constant and $r$ is the distance from the center of the shell. In addition, a small ball of charge $q = 45.0$ fC is located at that center. What value should $A$ have if the electric field in the shell ($a \le r \le b$) is to be uniform? (01小題)




$A =$__________ C/$m^2$

26: **ANS:=1.79e-11**

We use a Gaussian surface in the form of a sphere with radius $r_g$,

$$dV = 4\pi r^2\,dr\,.$$

$$q_s = 4\pi\int_a^{r_g} \rho r^2 dr = 4\pi \int_a^{r_g} \frac{A}{r} r^2 dr = 4\pi\, A \int_a^{r_g} r\,dr = 2\pi\, A\left(r_g^2 - a^2\right).$$

The total charge inside the Gaussian surface is $q + q_s = q + 2\pi\, A\left(r_g^2 - a^2\right)$.

$$\Phi = 4\pi r_g^2 E\,, \quad 4\pi\varepsilon_0 E r_g^2 = q + 2\pi\, A\left(r_g^2 - a^2\right). \quad E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_g^2} + 2\pi\, A - \frac{2\pi A a^2}{r_g^2}\right].$$

For the field to be uniform, the first and last terms in the brackets must cancel. They do if $q - 2\pi A a^2 = 0$ or $A = q/2\pi a^2$. With $a = 2.00 \times 10^{-2}$ m and $q = 45.0 \times 10^{-15}$ C, we have $A = 1.79 \times 10^{-11}\,\text{C/m}^2$.