

Problem 1

In the figure, find the equivalent capacitance of the combination. Assume that $C_1=10\mu\text{F}$, $C_2=5\mu\text{F}$, and $C_3=4\mu\text{F}$.

(01小題)

the equivalent capacitance = _____ μF

01: ANS:=3.16

$$C_{\text{eq}} = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3} = \frac{(10.0\mu\text{F} + 5.00\mu\text{F})(4.00\mu\text{F})}{10.0\mu\text{F} + 5.00\mu\text{F} + 4.00\mu\text{F}} = 3.16\mu\text{F}.$$

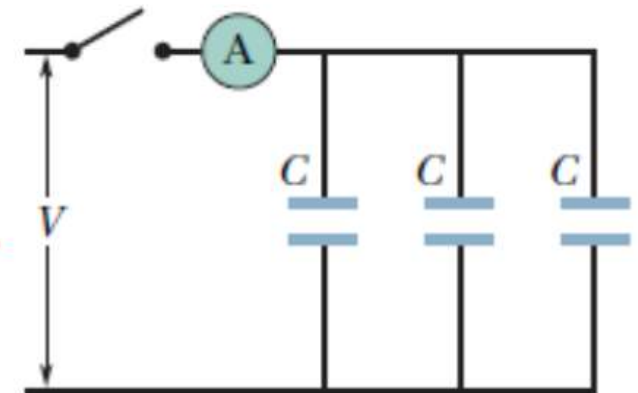
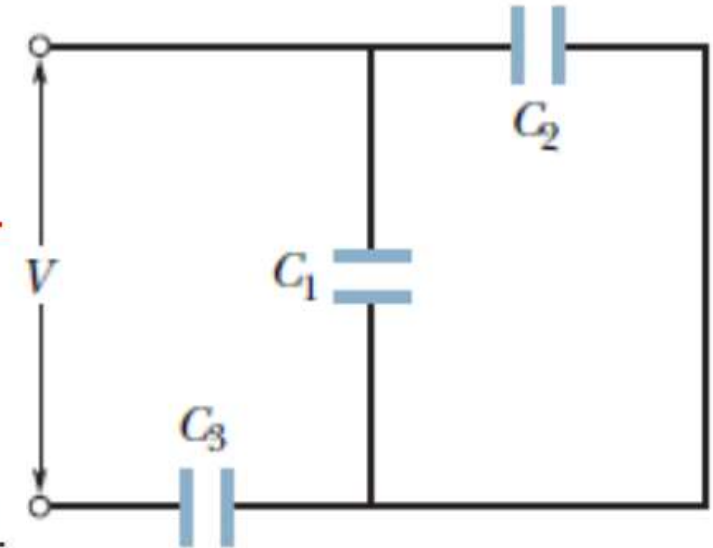
Each of the uncharged capacitors in the figure has a capacitance of $25\mu\text{F}$. A potential difference of $V = 4200\text{V}$ is established when the switch is closed. How many coulombs of charge then pass through meter A ?

(01小題)

How many coulombs of charge then pass through meter A = _____ C

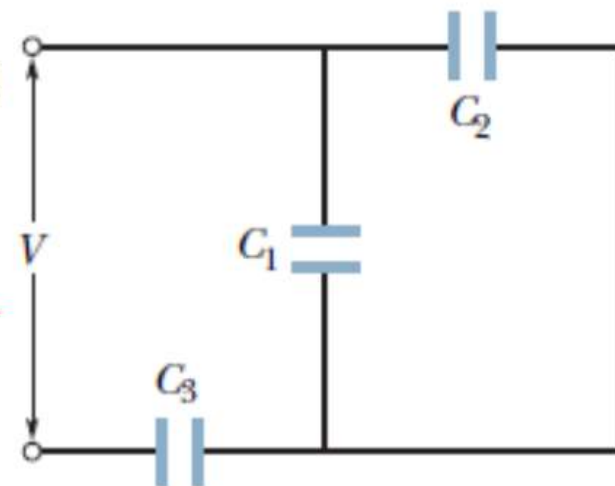
02: ANS:=0.315

$$q = C_{\text{eq}}V = 3CV = 3(25.0\mu\text{F})(4200\text{V}) = 0.315\text{C}.$$



Problem 2

In the figure, a potential difference of $V = 100 \text{ V}$ is applied across a capacitor arrangement with capacitances $C_1 = 10 \mu\text{F}$, $C_2 = 5 \mu\text{F}$, and $C_3 = 4 \mu\text{F}$. If capacitor 3 undergoes electrical breakdown so that it becomes equivalent to conducting wire, what is the increase in (a) the charge on capacitor 1 and (b) the potential difference across capacitor 1? (02小題)



(a) the charge on $C_1 = \underline{\hspace{2cm}} \text{ C}$

03: ANS: = 7.89E-4

(b) the potential difference across $C_1 = \underline{\hspace{2cm}} \text{ V}$

04: ANS: = 78.9

(a) and (b) The original potential difference V_1 across C_1 is

$$V_1 = \frac{C_{\text{eq}} V}{C_1 + C_2} = \frac{(3.16 \mu\text{F})(100.0 \text{ V})}{10.0 \mu\text{F} + 5.00 \mu\text{F}} = 21.1 \text{ V}.$$

Thus $\Delta V_1 = 100.0 \text{ V} - 21.1 \text{ V} = 78.9 \text{ V}$ and

$$\Delta q_1 = C_1 \Delta V_1 = (10.0 \mu\text{F})(78.9 \text{ V}) = 7.89 \times 10^{-4} \text{ C}.$$

Problem 2

A 100 pF capacitor is charged to a potential difference of 50 V, and the charging battery is disconnected. The capacitor is then connected in parallel with a second (initially uncharged) capacitor. If the potential difference across the first capacitor drops to 35 V, what is the capacitance of this second capacitor? (01小題)

the capacitance of this second capacitor=_____pF

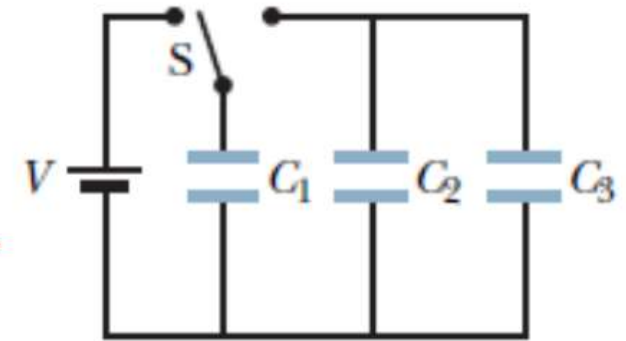
05: ANS:=43

The charge initially on the charged capacitor is given by $q = C_1 V_0$, where $C_1 = 100$ pF is the capacitance and $V_0 = 50$ V is the initial potential difference. After the battery is disconnected and the second capacitor wired in parallel to the first, the charge on the first capacitor is $q_1 = C_1 V$, where $V = 35$ V is the new potential difference. Since charge is conserved in the process, the charge on the second capacitor is $q_2 = q - q_1$, where C_2 is the capacitance of the second capacitor. Substituting $C_1 V_0$ for q and $C_1 V$ for q_1 , we obtain $q_2 = C_1 (V_0 - V)$. The potential difference across the second capacitor is also V , so the capacitance is

$$C_2 = \frac{q_2}{V} = \frac{V_0 - V}{V} C_1 = \frac{50 \text{ V} - 35 \text{ V}}{35 \text{ V}} (100 \text{ pF}) = 43 \text{ pF}.$$

Problem 3

In the figure, $V = 10 \text{ V}$, $C_1 = 10 \mu\text{F}$, and $C_2 = C_3 = 20 \mu\text{F}$. Switch S is first thrown to the left side until capacitor 1 reaches equilibrium. Then the switch is thrown to the right. When equilibrium is again reached, how much charge is on capacitor 1? (01小題)



How much charge is on $C_1 = \underline{\hspace{2cm}} \mu\text{F}$

06: ANS: = 20

Charge is conserved; $Q = q_1 + q_2 + q_3$.

C_2 and C_3 are identical, $q_2 = q_3$.

$$V_1 = V_3,$$

$$\frac{q_1}{C_1} = \frac{q_3}{C_3} \quad q_1 = q_3/2$$

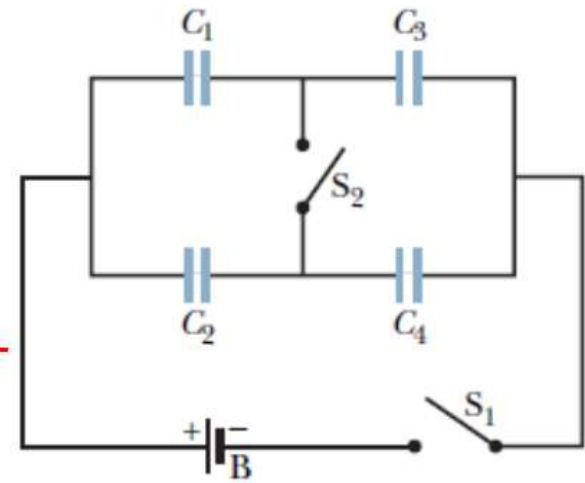
$$Q = (q_3/2) + q_3 + q_3 = 5q_3/2$$

$$q_3 = 2Q/5 = 2(100 \mu\text{C})/5 = 40 \mu\text{C}$$

$$q_1 = q_3/2 = 20 \mu\text{C}.$$

Problem 4

The figure shows a 12.0 V battery and four uncharged capacitors of capacitances $C_1=1\mu\text{F}$, $C_2=2\mu\text{F}$, $C_3=3\mu\text{F}$, and $C_4=4\mu\text{F}$. If only switch S_1 is closed, what is the charge on (a) capacitor 1, (b) capacitor 2, (c) capacitor 3, and (d) capacitor 4? If both switches are closed, what is the charge on (e) capacitor 1, (f) capacitor 2, (g) capacitor 3, and (h) capacitor 4? (08小題)



Only switch S_1 is closed:

(a) the charge on $C_1 = \underline{\hspace{2cm}} \mu\text{C}$

07: ANS: = 9

(b) the charge on $C_2 = \underline{\hspace{2cm}} \mu\text{C}$

08: ANS: = 16

(c) the charge on $C_3 = \underline{\hspace{2cm}} \mu\text{C}$

09: ANS: = 9

(d) the charge on $C_4 = \underline{\hspace{2cm}} \mu\text{C}$

10: ANS: = 16

Both switches S_1 and S_2 are closed:

(e) the charge on $C_1 = \underline{\hspace{2cm}} \mu\text{C}$

11: ANS: = 8.4

(f) the charge on $C_2 = \underline{\hspace{2cm}} \mu\text{C}$

12: ANS: = 16.8

(g) the charge on $C_3 = \underline{\hspace{2cm}} \mu\text{C}$

13: ANS: = 10.8

(h) the charge on $C_4 = \underline{\hspace{2cm}} \mu\text{C}$

14: ANS: = 14.4

(a) In this situation, capacitors 1 and 3 are in series, which means their charges are necessarily the same:

$$q_1 = q_3 = \frac{C_1 C_3 V}{C_1 + C_3} = \frac{(1.00 \mu\text{F})(3.00 \mu\text{F})(12.0 \text{V})}{1.00 \mu\text{F} + 3.00 \mu\text{F}} = 9.00 \mu\text{C}.$$

(b) Capacitors 2 and 4 are also in series:

$$q_2 = q_4 = \frac{C_2 C_4 V}{C_2 + C_4} = \frac{(2.00 \mu\text{F})(4.00 \mu\text{F})(12.0 \text{V})}{2.00 \mu\text{F} + 4.00 \mu\text{F}} = 16.0 \mu\text{C}.$$

(c) $q_3 = q_1 = 9.00 \mu\text{C}.$

(d) $q_4 = q_2 = 16.0 \mu\text{C}.$

(e) With switch 2 also closed, the potential difference V_1 across C_1 must equal the potential difference across C_2 and is

$$V_1 = \frac{C_3 + C_4}{C_1 + C_2 + C_3 + C_4} V = \frac{(3.00 \mu\text{F} + 4.00 \mu\text{F})(12.0 \text{V})}{1.00 \mu\text{F} + 2.00 \mu\text{F} + 3.00 \mu\text{F} + 4.00 \mu\text{F}} = 8.40 \text{V}.$$

Thus, $q_1 = C_1 V_1 = (1.00 \mu\text{F})(8.40 \text{V}) = 8.40 \mu\text{C}.$

(f) Similarly, $q_2 = C_2 V_1 = (2.00 \mu\text{F})(8.40 \text{V}) = 16.8 \mu\text{C}.$

(g) $q_3 = C_3(V - V_1) = (3.00 \mu\text{F})(12.0 \text{V} - 8.40 \text{V}) = 10.8 \mu\text{C}.$

(h) $q_4 = C_4(V - V_1) = (4.00 \mu\text{F})(12.0 \text{V} - 8.40 \text{V}) = 14.4 \mu\text{C}.$

Problem 5

What capacitance is required to store an energy of 10 kW·h at a potential difference of 1000 V? (01小題)

the capacitance=_____F

15: ANS:=72

$$U = \frac{1}{2} CV^2, \quad 10 \text{ kW} \cdot \text{h} = 3.6 \times 10^7 \text{ J}. \quad C = \frac{2U}{V^2} = \frac{2(3.6 \times 10^7 \text{ J})}{(1000 \text{ V})^2} = 72 \text{ F}.$$

The parallel plates in a capacitor, with a plate area of 8.50 cm² and an air-filled separation of 3.00 mm, are charged by a 6.00 V battery. They are then disconnected from the battery and pulled apart (without discharge) to a separation of 8.00 mm. Neglecting fringing, find (a) the potential difference between the plates, (b) the initial stored energy, (c) the final stored energy, and (d) the work required to separate the plates. (04小題)

(a) the potential difference between the plates=_____ V

16: ANS:=16.0

(b) the initial stored energy,

17: ANS:=4.51E-11

(c) the final stored energy,

18: ANS:=1.2E-10

d) the work required to separate the plates.

19: ANS:=7.52E-11

$$U_i = \frac{1}{2} CV_i^2 = \frac{\epsilon_0 AV_i^2}{2d} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(8.50 \times 10^{-4} \text{ m}^2)(6.00 \text{ V})^2}{2(3.00 \times 10^{-3} \text{ m})} = 4.51 \times 10^{-11} \text{ J}.$$

$$U_f = \frac{1}{2} \frac{\epsilon_0 A}{d_f} V_f^2 = \frac{1}{2} \frac{\epsilon_0 A}{d_f} \left(\frac{d_f}{d_i} V_i \right)^2 = \frac{d_f}{d_i} \left(\frac{\epsilon_0 AV_i^2}{2d_i} \right) = \frac{d_f}{d_i} U_i.$$

$$W = U_f - U_i = 7.52 \times 10^{-11} \text{ J}.$$

$$d_i = 3.00 \times 10^{-3} \text{ m},$$

$$V_i = 6.00 \text{ V}$$

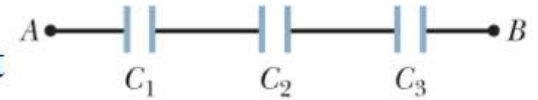
$$d_f = 8.00 \times 10^{-3} \text{ m},$$

$$U_f = 1.20 \times 10^{-10} \text{ J}.$$

$$V_f = 16.0 \text{ V}.$$

Problem 6

In the figure, $C_1 = 10.0\mu\text{F}$, $C_2 = 20.0\mu\text{F}$, and $C_3 = 25.0\mu\text{F}$. If no capacitor can withstand a potential difference of more than 100 V without failure, what are (a) the magnitude of the maximum potential difference that can exist between points A and B and (b) the maximum energy that can be stored in the three-capacitor arrangement? (02/小題)



(a) the potential difference = _____ V

20: ANS: = 190

(b) maximum energy = _____ J

21: ANS: = 0.095

(a) They each store the same charge, so the maximum voltage is across the smallest capacitor. With 100 V across $10\mu\text{F}$, then the voltage across the $20\mu\text{F}$ capacitor is 50 V and the voltage across the $25\mu\text{F}$ capacitor is 40 V. Therefore, the voltage across the arrangement is 190 V.

(b) we sum the energies on the capacitors and obtain $U_{\text{total}} = 0.095\text{ J}$.

Problem 7

Given a 7.4 pF air-filled capacitor, you are asked to convert it to a capacitor that can store up to 7.4 J with a maximum potential difference of 652 V. Which dielectric in Table 25-1 should you use to fill the gap in the capacitor if you do not allow for a margin of error? (01小題)

Find the dielectric constant. $\kappa =$ _____

22: ANS:=4.7

$$C = \kappa C_0,$$

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \kappa C_0 V^2,$$

$$\kappa = \frac{2U}{C_0 V^2} = \frac{2(7.4 \times 10^{-6} \text{ J})}{(7.4 \times 10^{-12} \text{ F})(652 \text{ V})^2} = 4.7.$$

you should use Pyrex.

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, $\kappa =$ unity.

Problem 8

A certain substance has a dielectric constant of 2.8 and a dielectric strength of 18 MV/m. If it is used as the dielectric material in a parallel-plate capacitor, what minimum area should the plates of the capacitor have to obtain a capacitance of $7 \times 10^{-2} \mu\text{F}$ and to ensure that the capacitor will be able to withstand a potential difference of 4.0 kV? (01小題)

the minimum area = _____ m^2

23: ANS: = 0.63

$$C = \kappa C_0 = \kappa \epsilon_0 A / d, \quad A = \frac{CV}{\kappa \epsilon_0 E}.$$

$$E = V/d,$$

$$d = V/E \text{ and } C = \kappa \epsilon_0 A E / V. \quad A = \frac{(7.0 \times 10^{-8} \text{ F})(4.0 \times 10^3 \text{ V})}{2.8(8.85 \times 10^{-12} \text{ F/m})(18 \times 10^6 \text{ V/m})} = 0.63 \text{ m}^2.$$

Problem 9

The figure shows a parallelplate capacitor with a plate area $A=7.89 \text{ cm}^2$ and plate separation $d=4.62 \text{ mm}$. The top half of the gap is filled with material of dielectric constant $\kappa_1=11$; the bottom half is filled with material of dielectric constant $\kappa_2=12$. What is the capacitance? (01小題)



the capacitance=_____pF

24: ANS:=17.3

$$E_1 = \frac{q}{\kappa_1 \epsilon_0 A}, \quad E_2 = \frac{q}{\kappa_2 \epsilon_0 A}.$$

$$V = \frac{E_1 d}{2} + \frac{E_2 d}{2} = \frac{qd}{2\epsilon_0 A} \left[\frac{1}{\kappa_1} + \frac{1}{\kappa_2} \right] = \frac{qd}{2\epsilon_0 A} \frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2},$$

$$C = \frac{q}{V} = \frac{2\epsilon_0 A}{d} \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}.$$

$$A = 7.89 \times 10^{-4} \text{ m}^2, \quad d = 4.62 \times 10^{-3} \text{ m}, \quad \kappa_1 = 11.0 \text{ and } \kappa_2 = 12.0,$$

$$C = \frac{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(7.89 \times 10^{-4} \text{ m}^2)(11.0)(12.0)}{4.62 \times 10^{-3} \text{ m} (11.0 + 12.0)} = 1.73 \times 10^{-11} \text{ F}.$$

Problem 10

A parallel-plate capacitor has a capacitance of 100 pF, a plate area of 100 cm², and a mica dielectric ($\kappa = 5.4$) completely filling the space between the plates. At 50 V potential difference, calculate (a) the electric field magnitude E in the mica, (b) the magnitude of the free charge on the plates, and (c) the magnitude of the induced surface charge on the mica.

(03小題)

(a) the electric field magnitude E in the mica, $E = \underline{\hspace{2cm}}$ N/C

25: ANS: = 10000

(b) the magnitude of the free charge on the plates, $Q = \underline{\hspace{2cm}}$ C

26: ANS: = 5E-9

(c) the magnitude of the induced surface charge, $Q = \underline{\hspace{2cm}}$ C

27: ANS: = 4.1E-9

$$E = V/d, \quad C = \kappa\epsilon_0 A/d, \quad d = \kappa\epsilon_0 A/C$$

$$E = \frac{VC}{\kappa\epsilon_0 A}$$

$$= \frac{(50 \text{ V})(100 \times 10^{-12} \text{ F})}{5.4(8.85 \times 10^{-12} \text{ F/m})(100 \times 10^{-4} \text{ m}^2)}$$

$$= 1.0 \times 10^4 \text{ V/m.}$$

(b) The free charge on the plates is $q_f = CV = (100 \times 10^{-12} \text{ F})(50 \text{ V}) = 5.0 \times 10^{-9} \text{ C}$.

The electric field is produced by both the free and induced charge.

$$E = \frac{q_f}{2\epsilon_0 A} + \frac{q_f}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A},$$

$$\begin{aligned} q_i &= q_f - \epsilon_0 A E = 5.0 \times 10^{-9} \text{ C} - (8.85 \times 10^{-12} \text{ F/m})(100 \times 10^{-4} \text{ m}^2)(1.0 \times 10^4 \text{ V/m}) \\ &= 4.1 \times 10^{-9} \text{ C} = 4.1 \text{ nC}. \end{aligned}$$