Problem 1

GPN2-L08

A proton traveling at $23°$ with respect to the direction of a magnetic field of strength 2.60 mT experiences a magnetic force of 6.5×10^{-17} N. Calculate (a) the proton's speed and (b) its kinetic energy in electron-volts. (02小題)

(a)the speed=______(m/s)

01: **ANS:=<u>4E5</u>**

(b)the kinetic energy=______(eV)

02: **ANS:=<u>835</u>**

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$v = \frac{F_B}{eBsin\phi} = \frac{6.5 \times 10^{-17} N}{(1.6 \times 10^{-19} C)(2.6 \times 10^{-3} T) sin 23^0} = 4 \times 10^5 m/s$$

(b)

$$K = \frac{1}{2} m v^2 = \frac{1}{2} (1.67 \times 10^{-27} kg)(4 \times 10^5 m/s)^2 = 1.34 \times 10^{-16} J$$

$$K = \frac{1.34 \times 10^{-16} J}{1.6 \times 10^{-19} J/eV} = 835 eV$$

Problem 1

An electron moves through a uniform magnetic field given by $\mathbf{B} = B_x\mathbf{i} + 3B_x\mathbf{j}$. At a particular instant, the electron as velocity $\mathbf{v} = (2\mathbf{i} + 4\mathbf{j})$ m/s and the magnetic force acting on it is $6.4 \times 10^{-19}\mathbf{k}$ N . Find B_x. (01小題)

B_x=________T

03: **ANS:=-2**

$$\vec{F} = q\vec{v} \times \vec{B} = q\begin{vmatrix} v_x & v_y \\ B_x & B_y \end{vmatrix} = q(v_xB_y - v_yB_x)\hat{k} = q(v_x(3B_x) - v_yB_x)\hat{k}$$

$$B_y = 3B_x \qquad q(3v_x - v_y)b_x = F_z$$

$$\vec{F} = F_z\hat{k} \qquad B_x = \frac{F_z}{q(3v_x - v_y)}$$

$$F_z = 6.4 \times 10^{-19}N$$

$$= \frac{6.4 \times 10^{-19}N}{(-1.6 \times 10^{-19}C)[3(2m/s) - 4m/s]}$$

$$= -2$$

Problem 2

An electron has an initial velocity of $12\mathbf{j} + 15\mathbf{k}$ km/s and a constant acceleration of $2 \times 10^{12}\,\hat{\imath}$ m/s^2 in a region in which uniform electric and magnetic fields are present. If $\mathbf{B} = 400\hat{\imath}\ \mu$T, find the electric field $\vec{E}$. (03小題)

$\mathbf{E}$=$E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k}$ V/m.

E_x=________

04: **ANS:=-11.4**

We apply $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = m_e\vec{a}$ to solve for $\vec{E}$:

E_y=________

05: **ANS:=-6**

$$\vec{E} = \frac{m_e\vec{a}}{q} + \vec{B} \times \vec{v}$$

E_z=________

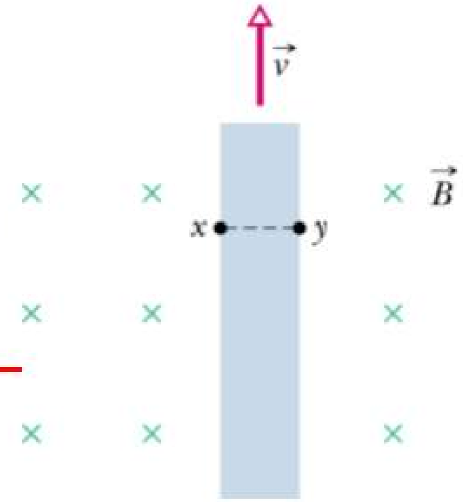
06: **ANS:=4.8**

$$= \frac{\left(9.11 \times 10^{-31}\,\text{kg}\right)\left(2.00 \times 10^{12}\,\text{m/s}^2\right)\hat{\text{i}}}{-1.60 \times 10^{-19}\,\text{C}} + (400\,\mu\text{T})\hat{\text{i}} \times \left[(12.0\,\text{km/s})\hat{\text{j}} + (15.0\,\text{km/s})\hat{\text{k}}\right]$$

$$= \left(-11.4\hat{\text{i}} - 6.00\hat{\text{j}} + 4.80\hat{\text{k}}\right)\text{V/m}.$$

Problem 3

A metal strip 6.50 cm long, 0.850 cm wide, and 0.760 mm thick moves with constant velocity $\vec{v}$ through a uniform magnetic field $B = 1.2$ mT directed perpendicular to the strip, as shown in the figure. A potential difference of $3.9\,\mu$V is measured between points x and y across the strip. Calculate the speed v. (01小題)



the speed v=______(m/s)

07: **ANS:=0.382**

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$v = \frac{E}{B} = \frac{(V_x - V_y)/d_{xy}}{B} = \frac{(V_x - V_y)}{Bd_{xy}} = \frac{3.9 \times 10^{-6} V}{(1.2 \times 10^{-3} T)(0.85 \times 10^{-2} m)} = 0.382 m/s$$

Problem 4

An electron of kinetic energy 1.20 keV circles in a plane perpendicular to a uniform magnetic field. The orbit radius is 25.0 cm. Find (a) the electron's speed, (b) the magnetic field magnitude, (c) the circling frequency, and (d) the period of the motion. (04小題)

(a)the electron's speed $v =$_____(m/s)

08: **ANS:=2.05E7**

(b)the magnetic field magnitude $B =$_____(T)

09: **ANS:=4.67E-4**

(c)the circling frequency $f =$_____

10: **ANS:=1.31E7**

(d)the period of the motion $T =$

11: **ANS:=7.63E-8**

$$K = \frac{1}{2} m_e v^2 \qquad v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(1.20\times10^3\,\text{eV})(1.60\times10^{-19}\,\text{eV/J})}{9.11\times10^{-31}\,\text{kg}}} = 2.05\times10^7\ \text{m/s}.$$

$$r = m_e v / qB \qquad B = \frac{m_e v}{qr} = \frac{(9.11\times10^{-31}\,\text{kg})(2.05\times10^7\ \text{m/s})}{(1.60\times10^{-19}\,\text{C})(25.0\times10^{-2}\,\text{m})} = 4.67\times10^{-4}\ \text{T}.$$

$$f = \frac{v}{2\pi r} = \frac{2.07\times10^7\ \text{m/s}}{2\pi(25.0\times10^{-2}\,\text{m})} = 1.31\times10^7\ \text{Hz}.$$

$$T = 1/f = (1.31\times10^7\ \text{Hz})^{-1} = 7.63\times10^{-8}\ \text{s}.$$

Problem 5

(a) Find the frequency of revolution of an electron with an energy of 100 eV in a uniform magnetic field of magnitude 35 μT. (b) Calculate the radius of the path of this electron if its velocity is perpendicular to the magnetic field. (02小題)

(a)the frequency, $f =$_____(Hz)

12: **ANS:=9.78E5**

$$f = \frac{Bq}{2\pi m_e} = \frac{(35.0\times10^{-6}\,\mathrm{T})(1.60\times10^{-19}\,\mathrm{C})}{2\pi(9.11\times10^{-31}\,\mathrm{kg})} = 9.78\times10^{5}\,\mathrm{Hz}.$$

(b)the radius, $r =$_____(m)

13: **ANS:=0.964**

$$r = \frac{m_e v}{qB} = \frac{\sqrt{2m_e K}}{qB} = \frac{\sqrt{2(9.11\times10^{-31}\,\mathrm{kg})(100\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{J/eV})}}{(1.60\times10^{-19}\,\mathrm{C})(35.0\times10^{-6}\,\mathrm{T})} = 0.964\,\mathrm{m}.$$

Problem 6

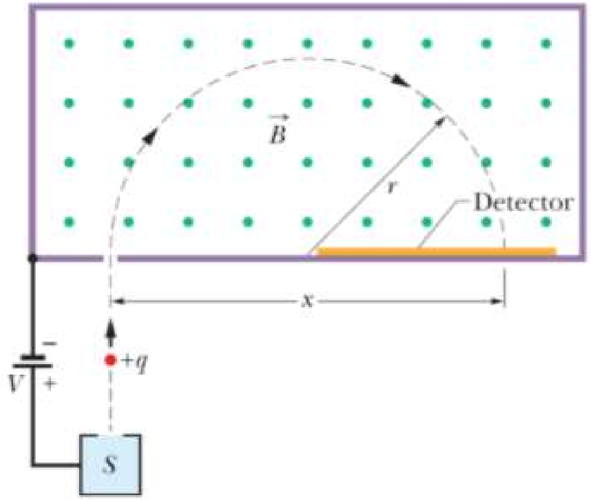
An electron follows a helical path in a uniform magnetic field of magnitude 0.300 T. The pitch of the path is 6.00 μm, and the magnitude of the magnetic force on the electron is 2.00×10^{-15} N. What is the electron's speed? (01小題)

speed v=______ m/s

14: **ANS:=6.53E4**

Problem 7

A certain commercial mass spectrometer is used to separate uranium ions of mass 3.92×10^{-25} kg and charge 3.20×10^{-19} C from related species. The ions are accelerated through a potential difference of 100 kV and then pass into a uniform magnetic field, where they are bent in a path of radius 1.00 m. After traveling through 180° and passing through a slit of width 1.00 mm and height 1.00 cm, they are collected in a cup. (a) What is the magnitude of the (perpendicular) magnetic field in the separator? If the machine is used to separate out 100 mg of material per hour, calculate (b) the current of the desired ions in the machine and (c) the thermal energy produced in the cup in 1.00 h. (03小題)



(a)magnetic field, $|B|$=________ T

15: **ANS:=0.495**

(b)the current, i=______ A

16: **ANS:=0.0227**

(c)thermal energy, Q=_______ J

17: **ANS:=8.17E6**

$K = \frac{1}{2}mv^2 = |\Delta U| = qV$

$x = 2r, \quad r = \frac{mv}{qB}$

$B = \frac{2mv}{qx} = \frac{2m}{qx}\sqrt{\frac{2qV}{m}}$

$= \sqrt{\frac{8Vm}{qx^2}} = \sqrt{\frac{8(100\times10^3)(3.92\times10^{-25})}{(3.2\times10^{-19})(2)^2}} = 0.495\ (T)$

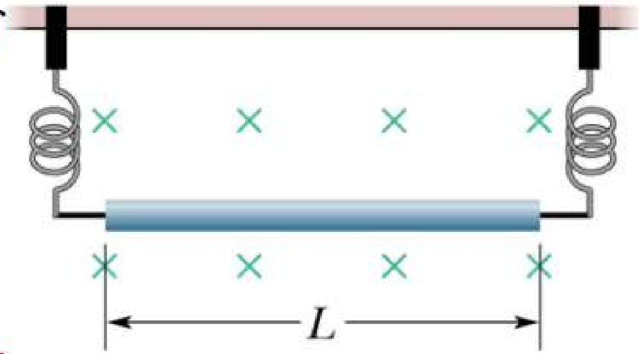
(b) $i = \frac{(3.2\times10^{-19})\left(\frac{100\times10^{-6}}{3600}\right)}{3.92\times10^{-25}} = 2.27\times10^{-2}\ (A)$

(c) $E = iV\Delta t = (2.27\times10^{-2})(100\times10^{3})(3600)$

$= 8.17\times10^{6}\ (J)$

Problem 8

A 13.0 g wire of length $L = 62$ cm is suspended by a pair of flexible leads in a uniform magnetic field of magnitude 0.440 T (The figure). What are the (a) magnitude and (b) direction (left or right) of the current required to remove the tension in the supporting leads? (02小題)



(a)the current=______(A)

18: **ANS:=0.467**

$$iLB = mg \quad \Rightarrow \quad i = \frac{mg}{LB} = \frac{(0.0130\,\text{kg})(9.8\,\text{m/s}^2)}{(0.620\,\text{m})(0.440\,\text{T})} = 0.467\,\text{A}.$$

(b)the direction? (left填 1, right填 2)

19: **ANS:=2**

the right-hand rule reveals that the current must be from left to right.

Problem 9

A wire 50.0 cm long carries a 0.500 A current in the positive direction of an x axis through a magnetic field $\vec{B} = 0.003\hat{j} + 0.01\hat{k}$ T. In unit-vector notation, what is the magnetic force on the wire? (03小題)

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}.$

F_x=_____ N

20: **ANS:=0**

F_y=_____ N

21: **ANS:=-2.5E-3**

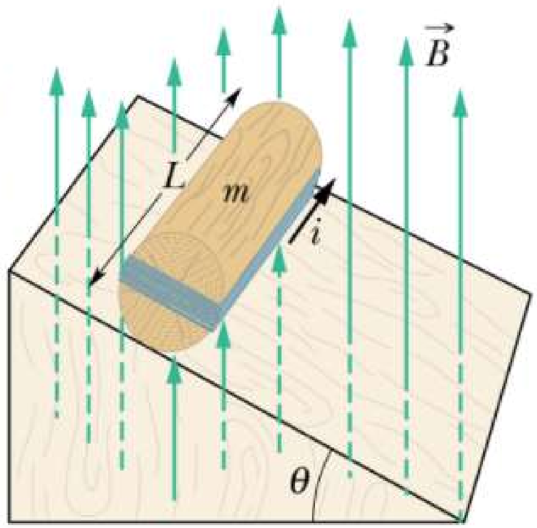
$$\vec{F} = i\vec{L} \times \vec{B}$$

F_z=_____ N

22: **ANS:=0.75E-3**

Problem 10

The figure shows a wood cylinder of mass $m = 0.25$ kg and length $L = 0.1$ m , with $N = 10$ turns of wire wrapped around it longitudinally, so that the plane of the wire coil contains the long central axis of the cylinder. The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the incline plane. If there is a vertical uniform magnetic field of magnitude 0.500 T, what is the least current i through the coil that keeps the cylinder from rolling down the plane? (01小題)



圓柱體維持靜態平衡必須合力為零，合力矩也為零，一個電流線圈在均勻的磁場中所受的磁力的合力為0，所以考慮合力的時候不需要考慮磁力，但是在合力矩考慮中就必須考慮電流所形成的磁矩在磁場中的力矩的貢獻。摩擦力對合力和合力矩都有貢獻，重力在垂直於斜面方向的分力與斜面的正向力互相抵銷不用考慮，重力沿著斜面往下的分力必須與沿著斜面往上的靜摩擦互相抵銷，才能使得合力為0，因此我們可以寫下下面公式：

$$mg\sin\theta - f = ma$$

$$fr - \mu B\sin\theta = I\alpha$$

$$a = 0 \text{ and } \alpha = 0,$$

$$mgr = \mu B.$$

$$\mu = NiA = Ni(2rL).$$

$$mgr = 2NirLB$$

$$i = \frac{mg}{2NLB} = \frac{(0.250\,\text{kg})(9.8\,\text{m/s}^2)}{2(10.0)(0.100\,\text{m})(0.500\,\text{T})} = 2.45\,\text{A}.$$

Problem 11

A circular coil of 160 turns has a radius of 1.90 cm. (a) Calculate the current that results in a magnetic dipole moment of magnitude **2.3 A · m²** . (b) Find the maximum magnitude of the torque that the coil, carrying this current, can experience in a uniform 35.0 mT magnetic field. (02小題)

(a)the current=______(A)

24: **ANS:=12.7**

$$\mu = NiA, \quad A = \pi r^2, \quad i = \frac{\mu}{N\pi r^2} = \frac{2.30\ \text{A}\cdot\text{m}^2}{(160)(\pi)(0.0190\ \text{m})^2} = 12.7\ \text{A}.$$

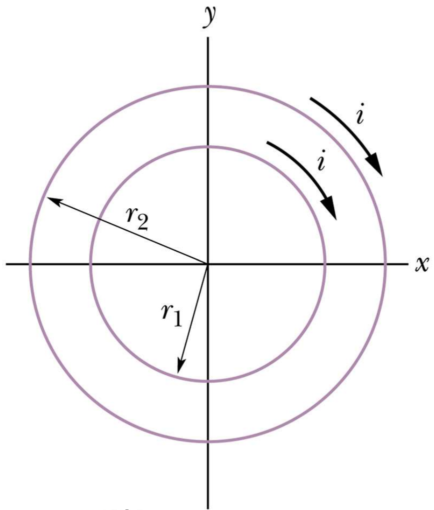
(b)the torque=______(N · m)

25: **ANS:=8.05E-2**

$$\tau_{\max} = \mu B = \left(2.30\ \text{A}\cdot\text{m}^2\right)\left(35.0\times10^{-3}\ \text{T}\right) = 8.05\times10^{-2}\ \text{N}\cdot\text{m}.$$

Problem 12

Two concentric, circular wire loops, of radii $r_1 = 20cm$ and $r_2 = 30cm$, are located in an xy plane; each carries a clockwise current of 7.00 A (the figure). (a) Find the magnitude of the net magnetic dipole moment of the system. (b) Repeat for reversed current in the inner loop. (02小題)



(a)μ=______ A · m²

26: **ANS:=2.86**

$$\mu = \sum_n i_n A_n = \pi r_1^2 i_1 + \pi r_2^2 i_2 = \pi(7.00\text{A})\left[(0.200\text{m})^2 + (0.300\text{m})^2\right] = 2.86\text{A}\cdot\text{m}^2.$$

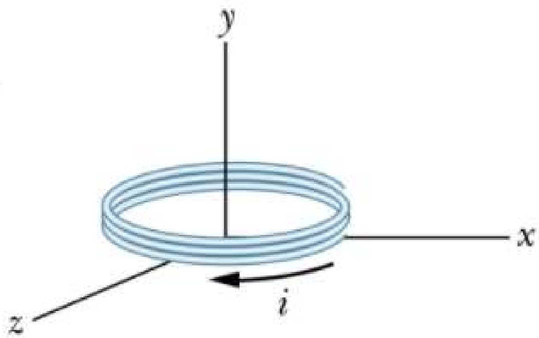
(b)μ=______ A · m²

$$\mu = \pi r_2^2 i_2 - \pi r_1^2 i_1 = \pi(7.00\text{A})\left[(0.300\text{m})^2 - (0.200\text{m})^2\right] = 1.10\text{A}\cdot\text{m}^2.$$

27: **ANS:=1.10**

Problem 13

The coil in the figure carries current $i = 2A$ in the direction indicated, is parallel to an xz plane, has 3.00 turns and an area of $4 \times 10^{-3}\ \text{m}^2$, and lies in a uniform magnetic field $\mathbf{B} = (2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$ mT. What are (a) the magnetic potential energy of the coil–magnetic field system and (b) the magnetic torque (in unit-vector notation) on the coil? (04小題)



(a) the magnetic potential energy =________J

28: **ANS:=-7.2E-5**

(b)$\vec{\tau} = A\hat{i} + B\hat{j} + C\hat{k}$(N \times m)

A=_____

29: **ANS:=9.6E-5**

$$\vec{\mu} = (NiA)(-\hat{j}) = -(0.0240\ \text{A}\cdot\text{m}^2)\hat{j}\,.$$

B=_____

30: **ANS:=0**

$$U = -\vec{\mu}\cdot\vec{B} = -(-0.0240\ \text{A}\cdot\text{m}^2)(-3.00\times10^{-3}\ \text{T}) = -7.20\times10^{-5}\ \text{J}\,.$$

C=_____

$$\hat{j}\times\hat{j} = 0,$$

31: **ANS:=4.8E-5**

$$\vec{\tau} = \vec{\mu}\times\vec{B} = (-0.0240\hat{j})\times(2.00\times10^{-3}\hat{i}) + (-0.0240\hat{j})\times(-4.00\times10^{-3}\hat{k})$$
$$= (4.80\times10^{-5}\hat{k} + 9.60\times10^{-5}\hat{i})\ \text{N}\cdot\text{m}.$$