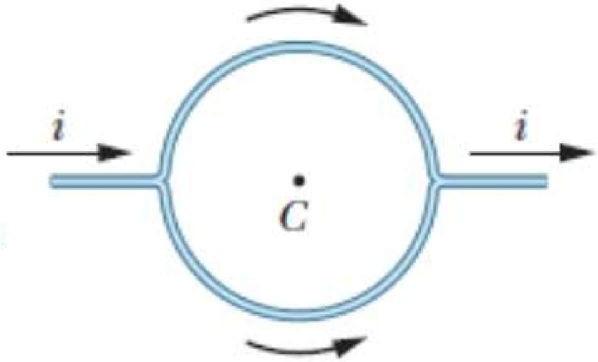
Problem 1

A straight conductor carrying current $i = 5.0A$ splits into identical semicircular arcs as shown in the figure. What is the magnetic field at the center C of the resulting circular loop? (01小題)



the magnetic field at the center, B=_____ [

01: **ANS:=B_C=_______T**

Solution:

The straight segment of the wire produces no magnetic field at C (see the straight sections discussion in Sample Problem 29-1). Also, the fields from the two semi-circular loops cancel at C (by symmetry). Therefore, $B_c = 0$.

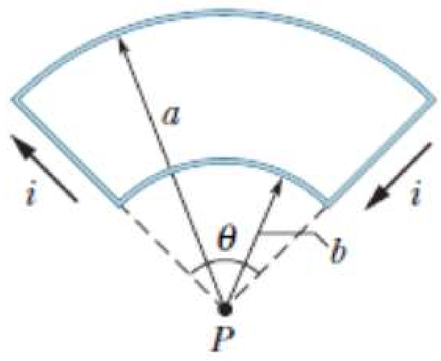
圓弧導線中，電流產生的磁場：$B = \dfrac{\mu_0 i\phi}{4\pi R}$

電流大小一樣，所以磁場大小一樣

但磁場方向相反。因此$B_c = 0$。

Problem 1

In the figure, two circular arcs have radii $a = 13.5cm$ and $b = 10.7cm$, subtend angle $\theta = 74.0^\circ$, carry current $i = 0.411A$, and share the same center of curvature P. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at P? (01小題)



(a)the magnitude B=________ [a,b,mu_0,i,phi]

02: **ANS:=(mu_0*i*phi)/(4*pi)*(1/b-1/a)**

Problem 1

Following previous problem, if a=13.5 cm, b=10.7 cm, $\theta = 74^\circ$, i=0.41 A. (02小題)

(b)the magnitude B=________T

03: **ANS:=1.02E-7**

(c) the direction=________into the page=1;out of the page=2

04: **ANS:=2**

Solution:

$$(a)B = \frac{\mu_0 i\phi}{4\pi}(\frac{1}{b} - \frac{1}{a})$$

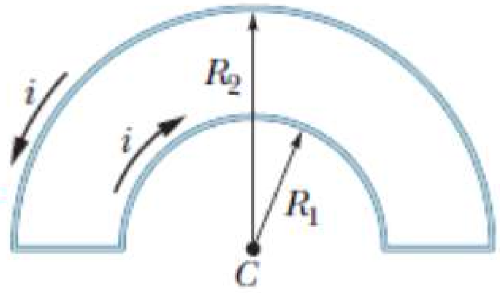
$$= \frac{(4\pi \times 10^{-7} T \cdot m/A)(0.411A)(\frac{74^0 \times \pi}{180^0})}{4\pi}(\frac{1}{0.107m} - \frac{1}{0.135m})$$

$$= 1.02 * 10^{-7} T$$

(b)The direction is out of the page.

Problem 2

In the figure, two semicircular arcs have radii $R_2 = 7.8cm$ and $R_1 = 3.15cm$, carry current $i = 0.281A$, and share the same center of curvature C. What are the (a) magnitude and (b) direction (into or out of the page) of the net magnetic field at C? (02小題)



(a) the magnitude B=_________T

05: **ANS:=1.67E-6**

(b) the direction=_________into the page=1;out of the page=2

06: **ANS:=1**

Solution:

$$(a) B = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$= \frac{(4\pi \times 10^{-7} T \cdot m/A)(0.281A)}{4}\left(\frac{1}{0.0315m} - \frac{1}{0.0780m}\right)$$
$$= 1.67 \times 10^{-6} T$$

(b)The direction of the field is into the page.

Problem 3

In the figure, a current $i = 10$ A is set up in a long hairpin conductor formed by bending a wire into a semicircle of radius $R = 5.0$ mm. Point b is midway between the straight sections and so distant from the semicircle that each straight section can be approximated as being an infinite wire. What are the (a) magnitude and (b) direction (into or out of the page) of $\vec{B}$ at a and the (c) magnitude and (d) direction of $\vec{B}$ at b? (04小題)

Magnetic field $\vec{B}$ at a:
(a)the magnitude B=________T

07: **ANS:=1E-3**

(b)the direction=________
into the page=1;out of the page=2

08: **ANS:=2**

Magnetic field $\vec{B}$ at b:
(c)the magnitude B=________T

09: **ANS:=8E-4**

(d)the direction=________
into the page=1;out of the page=2

10: **ANS:=2**

(a) $$B_a = 2\underbrace{\left(\frac{\mu_0 i}{4\pi R}\right)}_{\text{半長直導線}} + \underbrace{\frac{\mu_0 i \pi}{4\pi R}}_{\text{半圓}} = \frac{\mu_0 i}{2R}\left(\frac{1}{\pi}+\frac{1}{2}\right) = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(10\ \text{A})}{2(0.0050\ \text{m})}\left(\frac{1}{\pi}+\frac{1}{2}\right) = 1.0\times10^{-3}\ \text{T}$$

upon substituting $i = 10$ A and $R = 0.0050$ m.

PS：a 點可以視為在 2 條半無限長直導線中間位置的一點。

(b) The direction of this field is out of the page, as Fig. 29-6(c) makes clear.

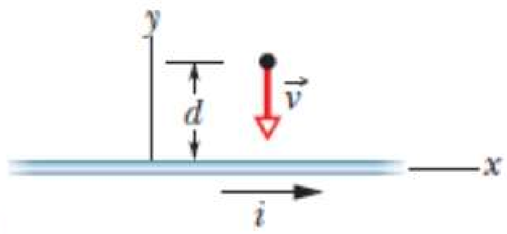
(c) The last remark in the problem statement implies that treating b as a point midway between two infinite wires is a good approximation. Thus,

$$B_b = 2\left(\frac{\mu_0 i}{2\pi R}\right) = \frac{\mu_0 i}{\pi R} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(10\ \text{A})}{\pi(0.0050\ \text{m})} = 8.0\times10^{-4}\ \text{T}.$$

(d) This field, too, points out of the page.

Problem 4

The figure shows a proton moving at velocity $\mathbf{v} = -200\hat{j}$ m/s toward a long straight wire with current $i = 350$ mA . At the instant shown, the proton's distance from the wire is $d = 2.89$ cm. In unit-vector notation, what is the magnetic force on the proton due to the current? (01小題)



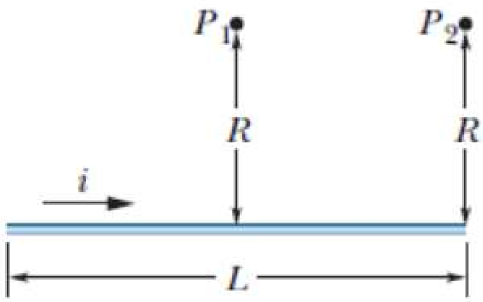
the magnitude of the force=________N

11: **ANS:=7.75E-23**

$$\mathbf{F} = \frac{\mu_0 i q v}{2\pi d}(-\mathbf{j} \times \mathbf{k})$$
$$= -\frac{\mu_0 i q v}{2\pi d}\hat{i}$$
$$= \frac{(4\pi \times 10^{-7} T \cdot m/A)(0.350A)(1.6 \times 10^{-19} C)(200 m/s)}{2\pi(0.0289m)}$$
$$= (-7.75 \times 10^{-23} N)\hat{i}$$

Problem 4

In the figure, point P_1 is at distance $R = 13.1$ cm on the perpendicular bisector of a straight wire of length $L = 18.0$ cm carrying current $i = 58.2$ mA . (Note that the wire is not long.) What is the magnitude of the magnetic field at P_1 due to i ? (01小題)



the magnitude of the magnetic field =__________T

12: **ANS:=5.03E-8**

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}dx \qquad B = \frac{\mu_0 i R}{4\pi}\int_{-L/2}^{L/2}\frac{dx}{(x^2+R^2)^{3/2}}$$

$$r = \sqrt{x^2+R^2} \qquad = \frac{\mu_0 i R}{4\pi}\frac{1}{R^2}\frac{x}{(x^2+R^2)^{1/2}}\Big|_{-L/2}^{L/2}$$

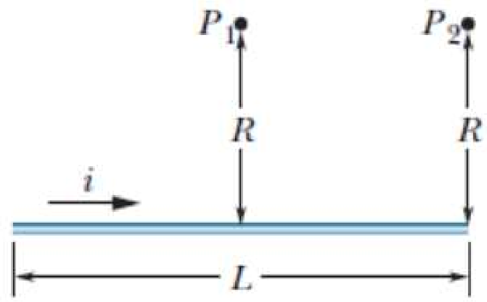
$$\sin\theta = \frac{R}{r} = \frac{R}{\sqrt{x^2+R^2}} \qquad = \frac{\mu_0 i}{2\pi R}\frac{L}{\sqrt{L^2+4R^2}}$$

$$= \frac{(4\pi\times10^{-7}T\cdot m/A)(0.0582A)}{2\pi(0.131m)}\frac{0.180m}{\sqrt{(0.180m)^2+4(0.131m)^2}}$$

$$= 5.03\times10^{-8}T$$

Problem 5

In the figure, point P_2 is at perpendicular distance $R = 25.1cm$ from one end of straight wire of length $L = 13.6$ cm carrying current $i = 0.693$ A. (Note that the wire is not long.) What is the magnitude of the magnetic field at P_2? (01小題)



the magnitude of the magnetic field=________T

13: **ANS:=1.32E-7**

$$dB = \frac{\mu_0 i}{4\pi}\frac{sin\theta}{r^2}dx$$

$$r = \sqrt{x^2 + R^2}$$

$$sin\theta = \frac{R}{r} = \frac{R}{\sqrt{x^2 + R^2}}$$

$$B = \frac{\mu_0 i R}{4\pi}\int_{-L}^{0}\frac{dx}{(x^2 + R^2)^{3/2}}$$

$$= \frac{\mu_0 i R}{4\pi}\frac{1}{R^2}\frac{x}{(x^2 + R^2)^{1/2}}\Big|_{-L}^{0}$$

$$= \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2 + R^2}}$$

$$= \frac{(4\pi \times 10^{-7}T \cdot m/A)(0.693A)}{4\pi(0.251m)}\frac{0.136m}{\sqrt{(0.136m)^2 + (0.251m)^2}}$$

$$= 1.32 \times 10^{-7}T$$

Problem 6

In the figure, two long straight wires (shown in cross section) carry currents $i_1 = 30.0$ mA and $i_2 = 40.0$ mA directly out of the page. They are equal distances from the origin, where they set up a magnetic field . To what value must current i_1 be changed in order to rotate 20.0° clockwise? (01小題)

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i_1=______ mA

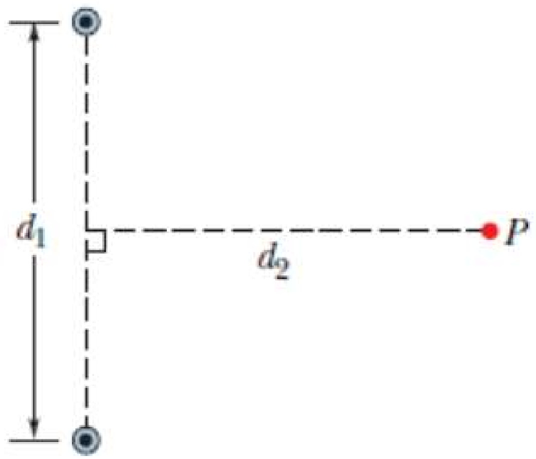
14: **ANS:=61.3**

$$\theta = \tan^{-1}(B_2/B_1) = \tan^{-1}(i_2/i_1) = 53.13^\circ.$$

$$\theta' = 53.13^\circ - 20^\circ = 33.13^\circ.$$

the current i_1 must be $i_2/\tan\theta' = 61.3$ mA.

The figure shows two very long straight wires (in cross section) that each carry a current of $4.00A$ directly out of the page. Distance $d_1 = 6.00$ m and distance $d_2 = 4.00$ m. What is the magnitude of the net magnetic field at point P, which lies on a perpendicular bisector to the wires? (01小題)



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the magnitude of the net magnetic field at point P=________T

15: **ANS:=2.56E-7**

Solution:

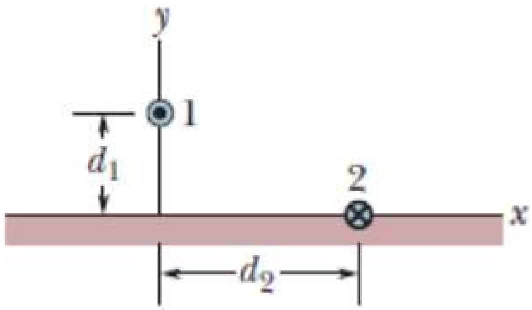
$$|\mathbf{B}| = 2\frac{\mu_0 i}{2\pi r}sin\theta \qquad |\mathbf{B}| = \frac{\mu_0 i}{\pi r}sin\theta$$

$$r = \sqrt{d_2^2 + (\frac{d_1}{2})^2} = \sqrt{(4m)^2 + (3m)^2} = 5m \qquad = \frac{(4\pi \times 10^{-7}T \cdot m/A)(4A)}{\pi(5m)}sin53.1^0$$

$$\theta = tan^{-1}\frac{d_2}{d_1/2} = tan^{-1}\frac{4m}{3m} = 53.1^0 \qquad = 2.56 \times 10^{-7}T$$

Problem 7

The figure shows wire 1 in cross section; the wire is long and straight, carries a current of 4.00 mA out of the page, and is at distance $d_1 = 2.40$ cm from a surface. Wire 2, which is parallel to wire 1 and also long, is at horizontal distance $d_2 = 5.00$ cm from wire 1 and carries a current of 6.80 mA into the page. What is the x component of the magnetic force per unit length on wire 2 due to wire 1? (01小題)



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the x component of the magnetic force per unit length=________N/m

16: **ANS:=8.84E-11**

$$cos\theta = \frac{d_2}{\sqrt{d_1^2 + d_2^2}}$$

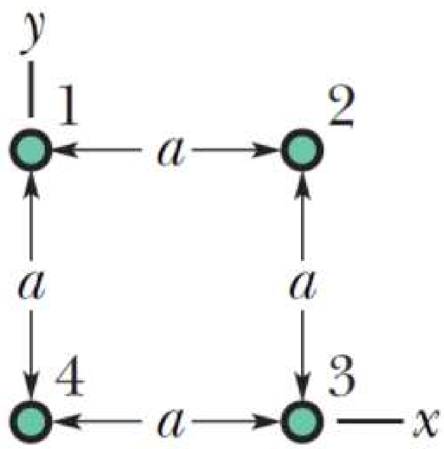
$$\frac{F_x}{L} = \frac{\mu_0 i_1 i_2 d_2}{2\pi(d_1^2 + d_2^2)}$$

$$= \frac{(4\pi \times 10^{-7} T \cdot m/A)(4 \times 10^{-3} A)(6.8 \times 10^{-3} A)(0.05m)}{2\pi[(0.024m)^2 + (0.050m)^2]}$$

$$= 8.84 \times 10^{-11} N/m$$

Problem 7

In the figure, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length a = 13.5 cm. Each wire carries 7.50 A, and the currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3. In unit-vector notation, what is the net magnetic force per meter of wire length on wire 4? $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$. (03小題)



(a) F_x=_____ N

17: **ANS:=-1.25E-4**

(b) F_y=_____ N

18: **ANS:=4.17E-5**

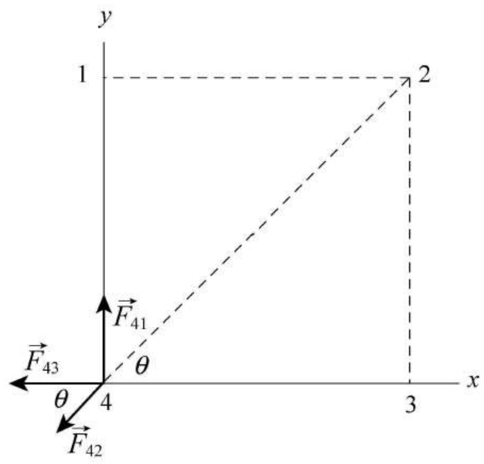
(c) F_z=_____ N

19: **ANS:=0**

$\vec{F}_4 = \vec{F}_{14} + \vec{F}_{24} + \vec{F}_{34}$. With $\theta = 45°$,

$$F_{4x} = -F_{43} - F_{42}\cos\theta = -\frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \cos 45°}{2\sqrt{2}\pi a} = -\frac{3\mu_0 i^2}{4\pi a}$$

$$F_{4y} = F_{41} - F_{42}\sin\theta = \frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \sin 45°}{2\sqrt{2}\pi a} = \frac{\mu_0 i^2}{4\pi a}.$$



$$F_4 = \left(F_{4x}^2 + F_{4y}^2\right)^{1/2} = \left[\left(-\frac{3\mu_0 i^2}{4\pi a}\right)^2 + \left(\frac{\mu_0 i^2}{4\pi a}\right)^2\right]^{1/2} = \frac{\sqrt{10}\mu_0 i^2}{4\pi a} = \frac{\sqrt{10}\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(7.50\mathrm{A})^2}{4\pi(0.135\mathrm{m})}$$

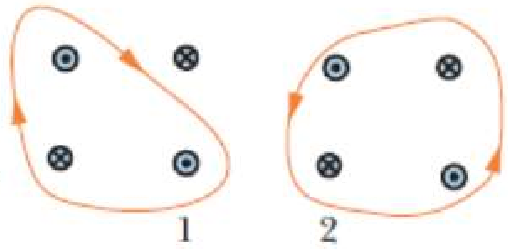
$$= 1.32\times10^{-4}\ \mathrm{N/m}$$

$\vec{F}_4$ makes an angle ϕ with the positive x axis $\phi = \tan^{-1}\left(\frac{F_{4y}}{F_{4x}}\right) = \tan^{-1}\left(-\frac{1}{3}\right) = 162°$.

$$\vec{F}_1 = (1.32\times10^{-4}\ \mathrm{N/m})[\cos 162°\hat{i} + \sin 162°\,\mathrm{j}] = (-1.25\times10^{-4}\ \mathrm{N/m})\hat{i} + (4.17\times10^{-5}\ \mathrm{N/m})\mathrm{j}$$

Problem 8

Each of the eight conductors in the figure carries 2.0 A of current into or out of the page. Two paths are indicated for the line integral $\oint \vec{B} \cdot d\vec{s}$. What is the value of the integral for (a) path 1 and (b) path 2? (02小題)



(a) the value of the integral for path 1=________$T \cdot m$

20: **ANS:=-2.5E-6**

(b) the value of the integral for path 2=________T$\cdot m$

21: **ANS:=0**

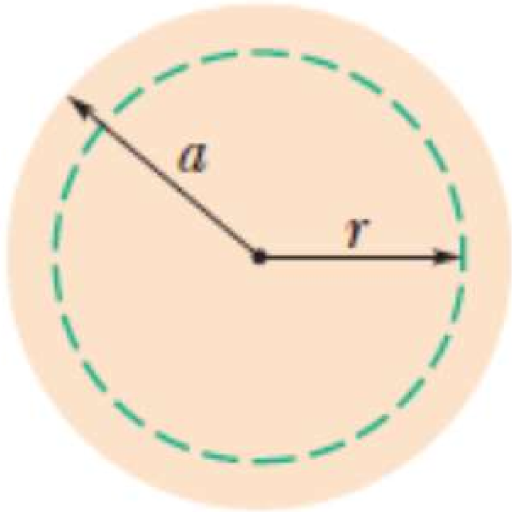
$$(a)\oint \vec{B} \cdot d\vec{S} = \mu_0 i$$

$$(b)\oint \vec{B} \cdot d\vec{S} = \mu_0 i_{enc} = 0$$

$$= -(4\pi \times 10^{-7} T \cdot m/A)(2A) = -2.5 \times 10^{-6} T \cdot m$$

Problem 9

The figure shows a cross section across a diameter of a long cylindrical conductor of radius a = 2.00 cm carrying uniform current 170 A. What is the magnitude of the current's magnetic field at radial distance (a) 0, (b) 1.00 cm, (c) 2.00 cm (wire's surface), and (d) 4.00 cm? (04小題)



(a) the magnitude of the current's magnetic field at radial distance 0 cm=_________T

22: **ANS:=<u>0</u>**

(b) the magnitude of the current's magnetic field at radial distance 1 cm=_________T

23: **ANS:=<u>8.5E-4</u>**

(c)at distance 2 cm, B=_________T

24: ANS:=1.7E-3

(d)at distance 4 cm, B=_________T

25: ANS:=8.5E-4

$B = \mu_0 ir / 2\pi a^2$ for the B-field inside the wire ($r < a$)

$B = \mu_0 i / 2\pi r$ for that outside the wire ($r > a$).

(a) At $r = 0$, $B = 0$.

(b) At $r = 0.0100\text{m}$, $B = \dfrac{\mu_0 ir}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0100\text{m})}{2\pi(0.0200\text{m})^2} = 8.50\times10^{-4}\,\text{T}.$

(c) At $r = a = 0.0200\text{m}$, $B = \dfrac{\mu_0 ir}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0200\text{m})}{2\pi(0.0200\text{m})^2} = 1.70\times10^{-3}\,\text{T}.$

(d) At $r = 0.0400\text{m}$, $B = \dfrac{\mu_0 i}{2\pi r} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})}{2\pi(0.0400\text{m})} = 8.50\times10^{-4}\,\text{T}.$

Problem 10

The current density $\vec{J}$ inside a long, solid, cylindrical wire of radius $a = 3.1$ mm is in the direction of the central axis, and its magnitude varies linearly with radial distance r from the axis according to $J = J_0 r/a$, where $J_0 = 310$ A/m^2. Find the magnitude of the magnetic field at (a) $r = 0$, (b) $r = a/2$, and (c) $r = a$. (03小題)

(a) $r = 0$, B=______ T

26: **ANS:=0**

(b) $r = a/2$, B=______ T

27: **ANS:=1E-7**

(c) $r = a$, B=______ T

28: **ANS:=4E-7**

For $r \le a$, $B(r) = \frac{\mu_0 i_{\text{enc}}}{2\pi r} = \frac{\mu_0}{2\pi r}\int_0^r J(r)\,2\pi r\,dr = \frac{\mu_0}{2\pi}\int_0^r J_0\left(\frac{r}{a}\right)2\pi r\,dr = \frac{\mu_0 J_0 r^2}{3a}$.

(a) $r = 0$, $B = 0$.

$$\text{(b) } r = a/2,\ B(r) = \frac{\mu_0 J_0 r^2}{3a} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(310\,\text{A/m}^2)(3.1\times10^{-3}\,\text{m}/2)^2}{3(3.1\times10^{-3}\,\text{m})} = 1.0\times10^{-7}\,\text{T}.$$

$$\text{(c) } r = a,\ B(r = a) = \frac{\mu_0 J_0 a}{3} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(310\,\text{A/m}^2)(3.1\times10^{-3}\,\text{m})}{3} = 4.0\times10^{-7}\,\text{T}.$$

Problem 11

A toroid having a square cross section, 5.00 cm on a side, and an inner radius of 15.0 cm has 500 turns and carries a current of 0.800 A. (It is made up of a square solenoid - instead of a round one -bent into a doughnut shape.) What is the magnetic field inside the toroid at (a) the inner radius and (b) the outer radius? (02小題)

(a) the magnetic field inside the toroid at the inner

29: **ANS:=5.33E-4**

(b) the magnetic field inside the toroid at the outer

30: **ANS:=4E-4**

$$B = \frac{\mu_0 iN}{2\pi r} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.800\,\text{A})(500)}{2\pi(0.150\,\text{m})} = 5.33\times10^{-4}\,\text{T}.$$

$$B = \frac{\mu_0 iN}{2\pi r} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.800\,\text{A})(500)}{2\pi(0.200\text{m})} = 4.00\times10^{-4}\,\text{T}.$$

Problem 11

A long solenoid with 10.0 turns/cm and a radius of 7.00 cm carries a current of 20.0 mA. A current of 6.00 A exists in a straight conductor located along the central axis of the solenoid. (a) At what radial distance from the axis will the direction of the resulting magnetic field be at 45.0° to the axial direction? (b) What is the magnitude of the magnetic field there? (02小題)

(a)the radial distance=_____ cm

31: **ANS:=4.77**

(b)the magnitude of the magnetic field=_____ T

32: **ANS:=3.55E-5**

$$\vec{B}_s \perp \vec{B}_w \quad B_s = B_w \quad B_s = \mu_0 i_s n = B_w = \frac{\mu_0 i_w}{2\pi d},$$

$$d = \frac{i_w}{2\pi i_s n} = \frac{6.00\,\text{A}}{2\pi\left(20.0\times10^{-3}\,\text{A}\right)\left(10\,\text{turns/cm}\right)} = 4.77\,\text{cm}.$$

$$B = \sqrt{2}B_s = \sqrt{2}\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(20.0\times10^{-3}\,\text{A}\right)\left(10\,\text{turns}/0.0100\,\text{m}\right) = 3.55\times10^{-5}\,\text{T}.$$