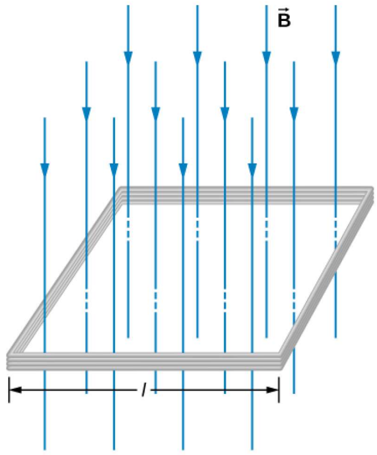
GPN2-LC10

Problem 1

The square coil of the figure has sides $l = 0.25$ m long and is tightly wound with $N = 200$ turns of wire. The resistance of the coil is $R = 5.0\,\Omega$. The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing at a rate $\frac{dB}{dt} = -0.04$ T/s. (a) What is the magnitude of the emf induced in the coil? (b) What is the magnitude of the current circulating through the coil? (02小題)



(a)the induced emf=______ V

01: **ANS:=<u>0.5</u>**

(b)the magnitude of the current=______ A

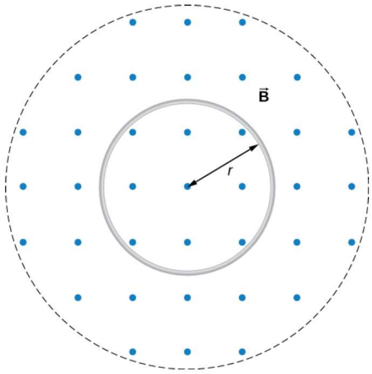
02: **ANS:=<u>0.1</u>**

Solution:

$$(a)|\mathscr{E}| = Nl^2\frac{dB}{dt} = (200)(0.25^2)(0.04) = 0.5$$

$$(b)I = \frac{(E)}{R} = \frac{0.5}{5} = 0.1$$

Problem 2



A magnetic field $\vec{B}$ is directed outward perpendicular to the plane of a circular coil of radius 0.50 m (see figure). The field is cylindrically symmetrical with respect to the center of the coil, and its magnitude decays exponentially according to \(B(t)=1.5 e^{-5t\), where B is in teslas and t is in seconds. (a) Calculate the emf $|\mathscr{E}|$ induced in the coil at the times $t=0.05$ s and (b) Determine the current I in the coil at $t=0.05$ if its resistance is 10Ω.(c)What is the induced electric field and (d)its direction in the circular coil at $t=0.05$ s. (04小題)

(a)$\mathscr{E}$=______ V

03: **ANS:=4.7**

(b)I=_____ A

04: **ANS:=0.47**

(c)the induced electric field, E=_______

05: **ANS:=1.5**

(d)the direction of the induced electric field =________ 1=clockwise; 2=counterclockwise.

06: **ANS:=2**

Problem 2

The current through the windings of a solenoid with $n = 200$ turns per meter is changing at a rate $\frac{dI}{dt} = 3.0$ A/s. The solenoid is 50-cm long and has a cross-sectional diameter of 3.0 cm. A small coil consisting of $N = 20$ closely wound turns wrapped in a circle of diameter 1.0 cm is placed in the middle of the solenoid such that the plane of the coil is perpendicular to the central axis of the solenoid. Assuming that the infinite-solenoid approximation is valid at the location of the small coil, determine the magnitude of the emf $\mathscr{E}$ induced in the coil. (01小題)

$\mathscr{E}$=_____ V

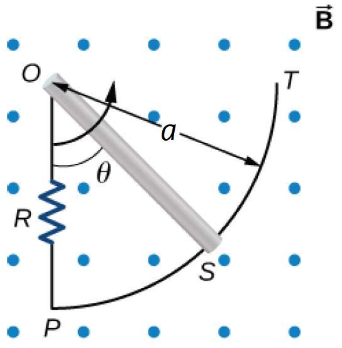
07: **ANS:=1.2E-5**

Solution:

$$\Phi_m = \mu_0 n I\left(\frac{\pi d^2}{4}\right)$$

$$\mathscr{E} = \left|N\frac{d\Phi_m}{dt}\right| = \left|N\mu_0 n\frac{\pi d^2}{4}\frac{dI}{dt}\right| = 20(4\pi\times10^{-7})(2000)\frac{\pi(0.01^2)}{4}(3.0) = 1.2\times10^{-5}$$

Problem 3



The figure shows a metal rod OS that is rotating in a horizontal plane around point O. The rod slides along a wire that forms a circular arc PST of radius r. The system is in a constant magnetic field that is directed out of the page. (a) If you rotate the rod at a constant angular velocity w, what is the current I in the closed loop OPSO? Assume that the resistor R furnishes all of the resistance in the closed loop. (b) Calculate the work per unit time that you do while rotating the rod and show that it is equal to the power dissipated in the resistor. (02小題)

(a)I=_____ [B,a,w,R]

08: **ANS:=(B*a**2*w/(2*R)**

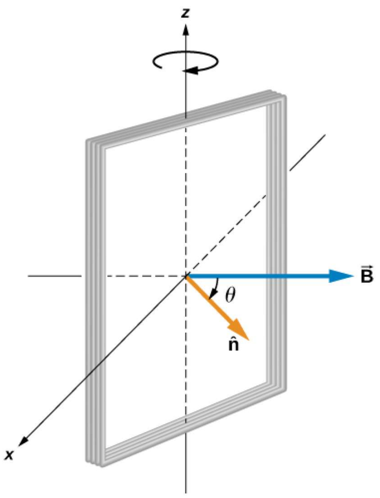
(b)P=______ [B,a,w,R]

09: **ANS:=(B**2*a**4*w**2)/(4*R)**

nprob= 3 3

Problem 3

A rectangular coil of area A and N turns is placed in a uniform magnetic field $\vec{B} = B\hat{j}$ as shown in the figure. The coil is rotated about the z-axis through its center at a constant angular velocity w. Obtain an expression for the induced emf (E) in the coil. (01小題)



emf (E)=_____ [w,N,A,B,t]

10: **ANS:= N*B*A*w*sin(w*t)**

Solution:

$\theta = wt; \Phi = BA\cos\theta;$

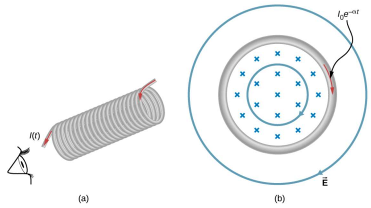
$$\mathscr{E} = -N\frac{d\Phi}{dt} = NBA\frac{d\theta}{dt}\sin\theta$$

$\mathscr{E} == NBAw\sin wt$

nprob= 4 4

Problem 4

Part (a) of the figure shows a long solenoid with radius R and n turns per unit length; its current decreases with time according to $I(t) = I_0 e^{-\alpha t}$. What is the magnitude of the induced electric field at a point a distance r from the central axis of the solenoid (a) when $r > R$ and (b) when $r < R$[see part (b) of the figure]. (c) What is the direction of the induced field at $r < R$? Assume that the infinite-solenoid approximation is valid throughout the regions of interest. (03小題)



(a)$r < R$; $\mathscr{E}$=______ [alpha,mu_0,n,I_0,R,r,t]

11: **ANS:=(alpha*mu_0*n*I_0*R**2)/(2*r)*exp(-alpha*t)**

(b)$r > R$; $\mathscr{E}$=______ [alpha,mu_0,n,I_0,R,r,t]

12: **ANS:=(alpha*mu_0*n*I_0*r)/2*exp(-alpha*t)**

(c)the direction of the induced field at $r < R$=_____ 1=clockwise; 2=counterclockwise.

13: **ANS:=1**

Solution:

(a)The magnetic field is confined to the interior of the solenoid:

$$B = \mu_0 n I = \mu_0 n I_0 e^{-\alpha t}$$

$$\Phi = BA = \mu_0 n I = \mu_0 n I_0 (\pi R^2) e^{-\alpha t}$$

$$\left| \oint \vec{E} \cdot d\vec{l} \right| = \mathscr{E} = \left| \frac{d\Phi}{dt} \right|$$

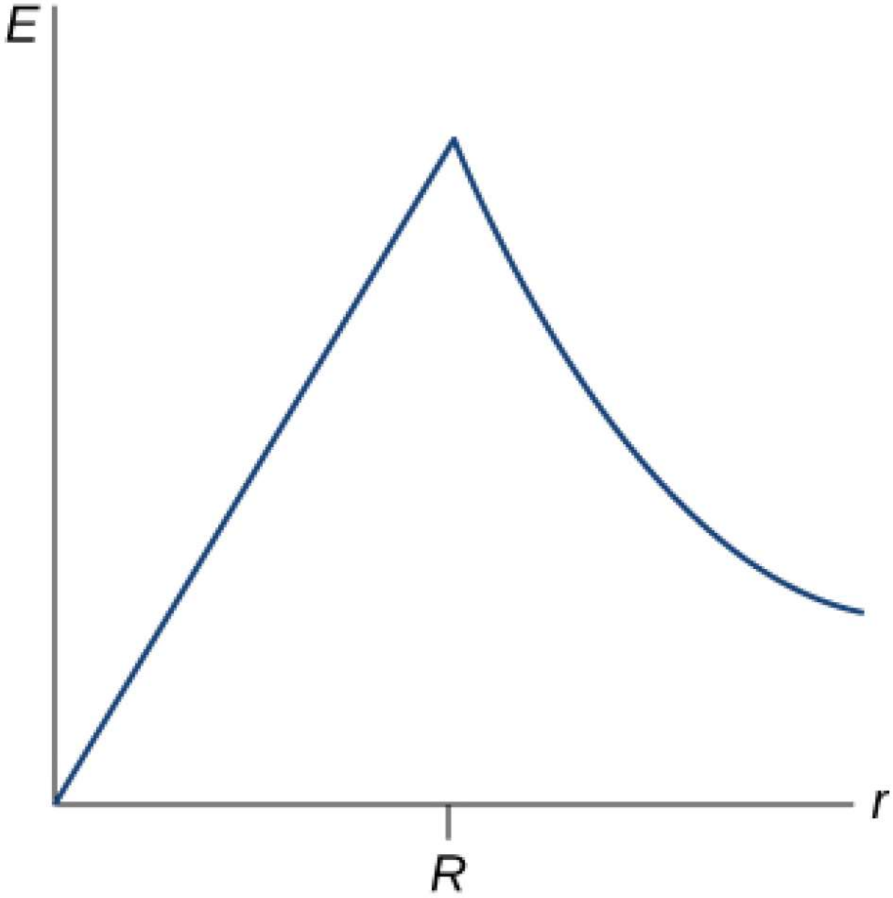
$$E(2\pi r) = \alpha \mu_0 n I_0 \pi R^2 e^{-\alpha t}$$

$$E = \frac{\alpha \mu_0 n I_0 R^2}{2r} e^{-\alpha t} \quad (r > R)$$

(b)$r < R$; $E(2\pi r) = \alpha \mu_0 n I_0 \pi r^2 e^{-\alpha t}$

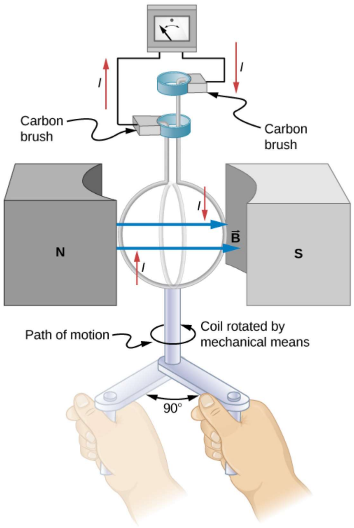
$$E = \frac{\alpha \mu_0 n I_0 r}{2} e^{-\alpha t}$$

(c) 磁場指向頁面，如 Fig.(b) 部分所示，並且正在減小。 如果任一圓形路徑被導電環佔據，則其中感應出的電流將如圖所示循環，符合楞次定律。 感應電場也必須如此定向，也就是順時針方向。


E
r
R

Problem 4

The generator coil shown in the figure is rotated through one-fourth of a revolution (from $\theta = 0^\circ$ to 90° in 15.0 ms. The 200-turn circular coil has a 5.00-cm radius and is in a uniform 0.80-T magnetic field. What is the emf induced? (01小題)



$\mathscr{E}$=_________ V

14: **ANS:=131**

Solution:

$$N = 200,\ B = 0.8,\ d\theta = \pi/2,\ dt = 0.015,$$

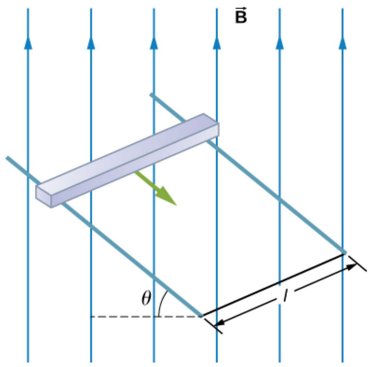
$$A = \pi r^2 = 7.85 \times 10^{-3}$$

$$\mathscr{E} = NBA \sin\theta \frac{d\theta}{dt}$$

$$\mathscr{E} = (200)(0.8)(7.85 \times 10^{-3}) \sin(\pi/2) \frac{\pi/2}{0.015} = 131$$

Problem 5

A square bar of mass m and resistance R is sliding without friction down very long, parallel conducting rails of negligible resistance (see below). The two rails are a distance l apart and are connected to each other at the bottom of the incline by a zero-resistance wire. The rails are inclined at an angle θ, and there is a uniform vertical magnetic field $\vec{B}$ throughout the region. (a)Find the terminal velocity of the bar; (b) Calculate the work per unit time done by the force of gravity. (02小題)



(a)the terminal velocity v=_______ [m,g,R,theta,B,l]

15: **ANS:=(m*g*R*sin(theta))/(B**2*l**2*cos(theta)**2)**

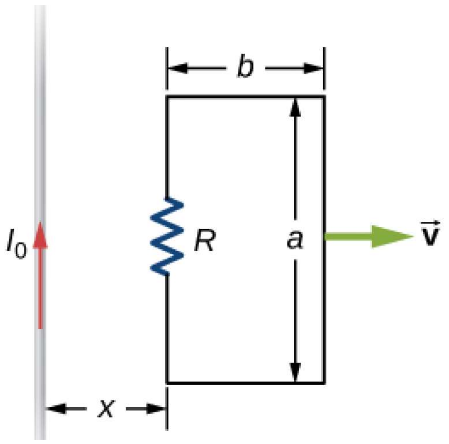
(b)the work per unit time done by the force of gravity=_____[m,g,R,theta,B,l]

16: **ANS:=m*g*v*sin(theta)**

nprob= 5 5

Problem 5

A rectangular circuit containing a resistance R is pulled at a constant velocity $\vec{v}$ away from a long, straight wire carrying a current I_0 (see figure). Derive an equation that gives the current induced in the circuit as a function of the distance x between the near side of the circuit and the wire. (02小題)



(a)$\mathscr{E}$=______ [mu_0,I_0,a,b,v,x]

17: **ANS:=(mu_0*I_0*a*b*v)/(2*pi*x*(x+b))**

(b)I=_______ [mu_0,I_0,a,b,v,x]

18: **ANS:=(mu_0*I_0*a*b*v)/(2*pi*R*x*(x+b))**

nprob= 6 6

Problem 6

Consider a long, cylindrical solenoid with lengthl, cross-sectional area A, and N turns of wire. What is the self-inductance of the solenoid? (02小題)

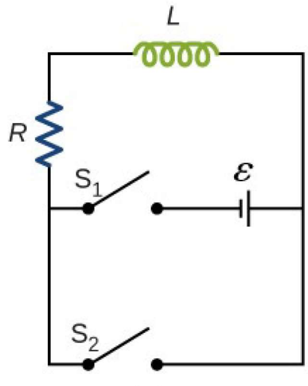
(a)the self-inductance, L=______ [mu_0,l,N,A]

19: **ANS:=mu_0*(N/L)**2*(A*l)**

(b)If the current in the solenoid is I, the magnetic energy stored in the solenoid=______ [mu_0,l,N,A,I]

20: **ANS:=1/2*mu_0*(N/L)**2*(A*l)*I**2**

Problem 7



In the circuit of the figure, let $\mathscr{E}=2.0\,V,\ \ R=4.0\,\Omega, L=4.0\,H$. With S_1 closed and S_2 open (see figure). (a) what is the time constant of the circuit? (b) What are the current in the circuit and the magnitude of the induced emf across the inductor at $t=0,\ t=2\tau_L,\ t=\infty$? (03小題)

(a)the time constant of the circuit, τ_L=______ s

21: **ANS:=<u>1.0</u>**

(b)at $t=2\tau_L,\ \ I$=_______ A

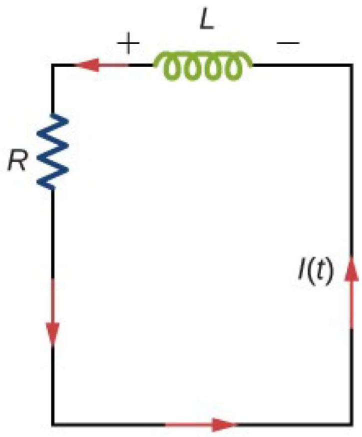
22: **ANS:=<u>0.43</u>**

(c)at $t=2\tau_L,\ \ V$=_______ V

23: **ANS:=<u>0.27</u>**

Problem 7

After the current in the RL circuit in previous problem has reached its final value, the positions of the switches are reversed so that the circuit becomes the one shown in the figure. (a) How long does it take the current to drop to half its initial value? (b) How long does it take before the energy stored in the inductor is reduced to of 1% of its maximum value? (02小題)



(a)current drops to half its initial value, $t_{1/2}$=_____ s

24: **ANS:=<u>0.69</u>**

(b)reduced to of 1% of its maximum magnetic energy, t=______ s

25: **ANS:=<u>2.3</u>**